Antenna Toolbox™

User's Guide
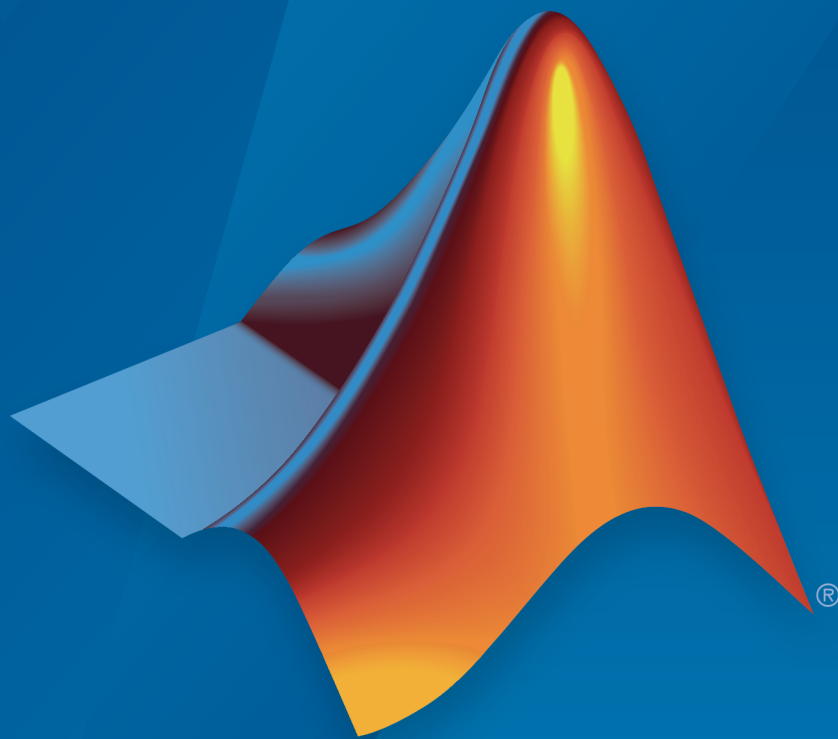
MATLAB®

MathWorks®

How to Contact MathWorks

	Latest news:	www.mathworks.com
	Sales and services:	www.mathworks.com/sales_and_services
	User community:	www.mathworks.com/matlabcentral
	Technical support:	www.mathworks.com/support/contact_us
	Phone:	508-647-7000

The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098

Contents

Antenna Concepts

1

2 Array Concepts

3 Computational Techniques

Antenna Concepts

Port Analysis

In this section...
"Input Impedance" on page 1-3
"Resonance" on page 1-5
"Reflection Coefficient" on page 1-6
"Return Loss" on page 1-7
"Voltage Standing Wave Ratio" on page 1-8
"Bandwidth" on page 1-10

The *port* of an antenna is the physical location where the RF source is connected. From a network theory perspective, the antenna has a single port. In Antenna Toolbox™, a red dot on the antenna figure represents the feed point. A half-wavelength dipole is shown with its feed point:
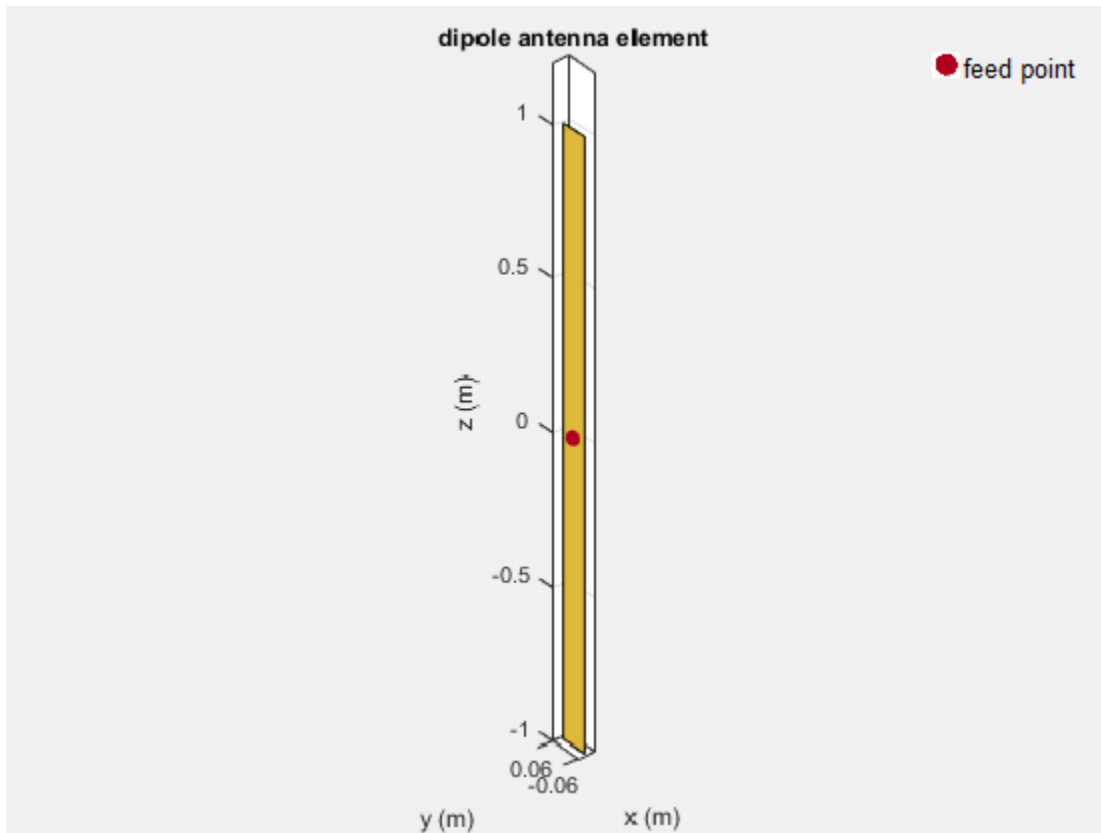
All antennas are excited by a voltage of 1V at the port. The various terminal port parameters are as follows:

Input Impedance

Input impedance is the ratio of voltage to current at the port. Antenna impedance is calculated as the ratio of the phasor voltage, which is 1V at a phase angle of 0 deg, to the phasor current at the port. The impedance is given by the equation:

$$Z = V / I = R + jX$$

where:

- `V` is the antenna excitation voltage
- `I` is the current
- `R` is the antenna resistance in ohms
- `X` is the antenna reactance in ohms

Antenna input impedance is a frequency-dependent quantity. The plot shows the input impedance of a dipole antenna over the frequency band 20–120 MHz. The resistance and reactance traces vary with frequency. The variation can be qualitatively described in terms of resonances.

```
d=dipole;
impedance(d,20e6:1e6:120e6)
```
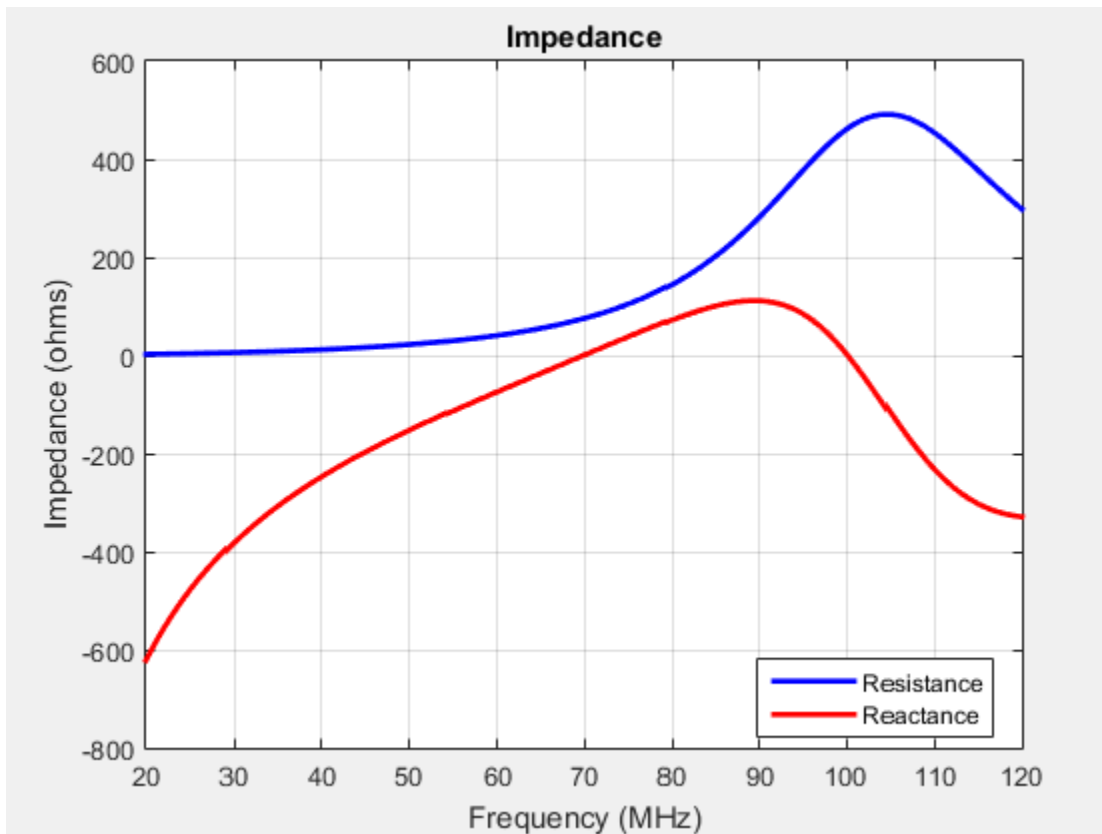
Use `impedance` to calculate the input impedance of any antennas in Antenna Toolbox.

Resonance

`

The *resonant frequency* of the antenna is the frequency at which the reactance of the antenna is equal to zero.

The plot shows two resonance points of a dipole antenna.
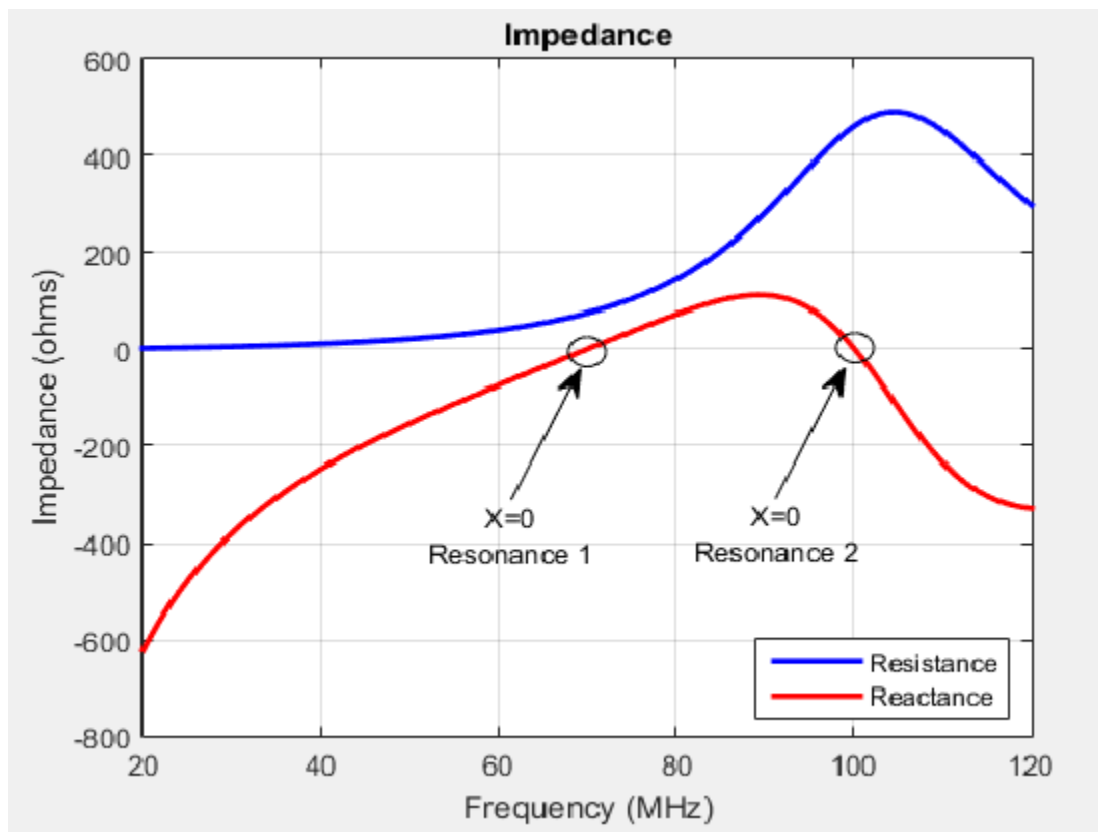
In the plot, the reactance values are negative, or capacitive, before the resonance. These values are positive or inductive after the resonance. This type of resonance is called series resonance, is modelled by a series RLC circuit. If the impedance curve goes from

positive reactance to negative reactance, it is called parallel resonance. This type of resonance is modelled by a parallel RLC circuit.

Reflection Coefficient

The *reflection coefficient*, or S_1_1, of the antenna describes a relative fraction of the incident RF power that is reflected back due to the impedance mismatch. Impedance mismatch is the difference between the input impedance of the antenna and the characteristic impedance of the transmission line (or the generator impedance when the transmission line is not present). The characteristic impedance is the reference impedance.

```
S = sparameters(d,20e6:1e6:120e6,72)
rfplot(S)
```
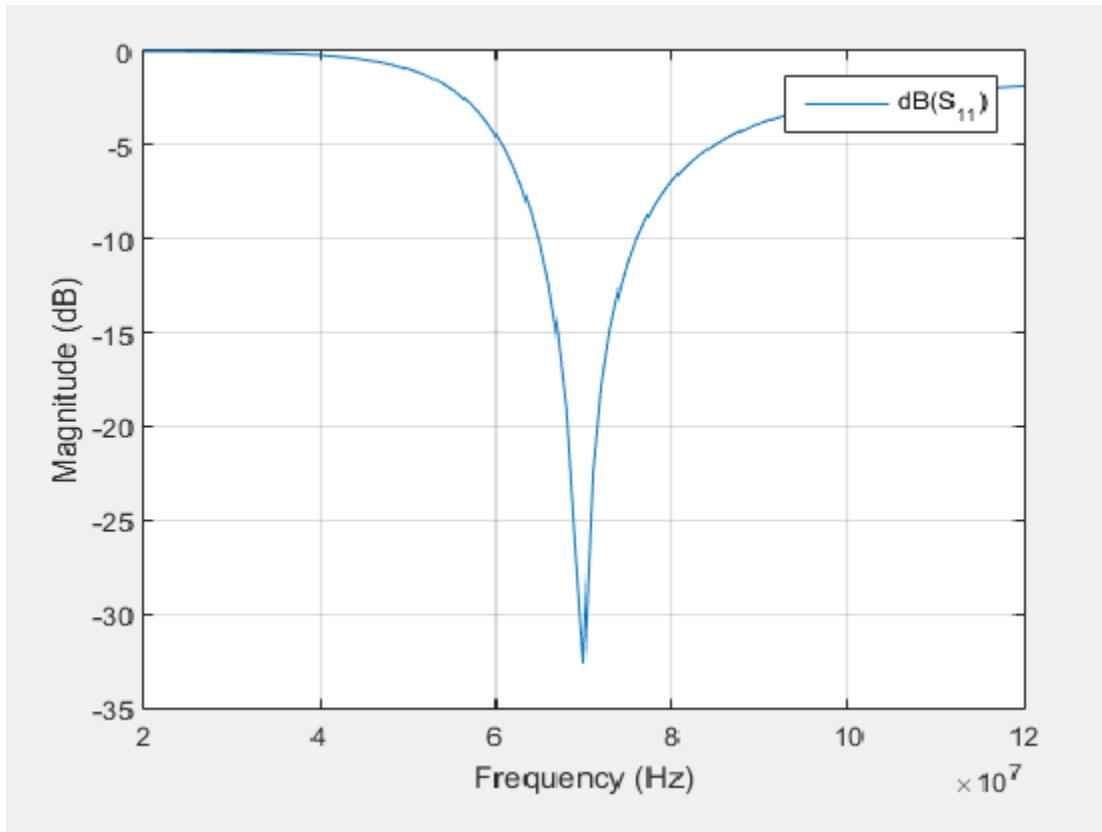
The reflection coefficient also gives the operating bandwidth of the antenna. Antenna bandwidth is usually the frequency band over which the magnitude of the reflection coefficient is below –10 dB.

Use `sparameters` to calculate the value of S_{11} for any antenna in the Antenna Toolbox.

Return Loss

The *return loss* of an antenna is a measure of the effectiveness of power delivery from a transmission line or coaxial cable to a load such as an antenna. The return loss can also be defined as the difference in dB between the power sent toward the antenna and the power reflected back from it. The higher the power ratio, the better matching between load and line. Return loss is given by the equation:

$$RL = -20\log_{10}\left|S_{11}\right|$$

where:

- `RL` is the return loss
- S_{11} is the reflection coefficient, or power reflected from the antenna.

For passive devices, the return loss is a positive nondissipative term representing the reduction in amplitude of the reflected wave in comparison to the incident wave. In active devices, a negative return loss is possible.

```
d=dipole;
returnLoss(d,20e6:1e6:120e6,72)
```
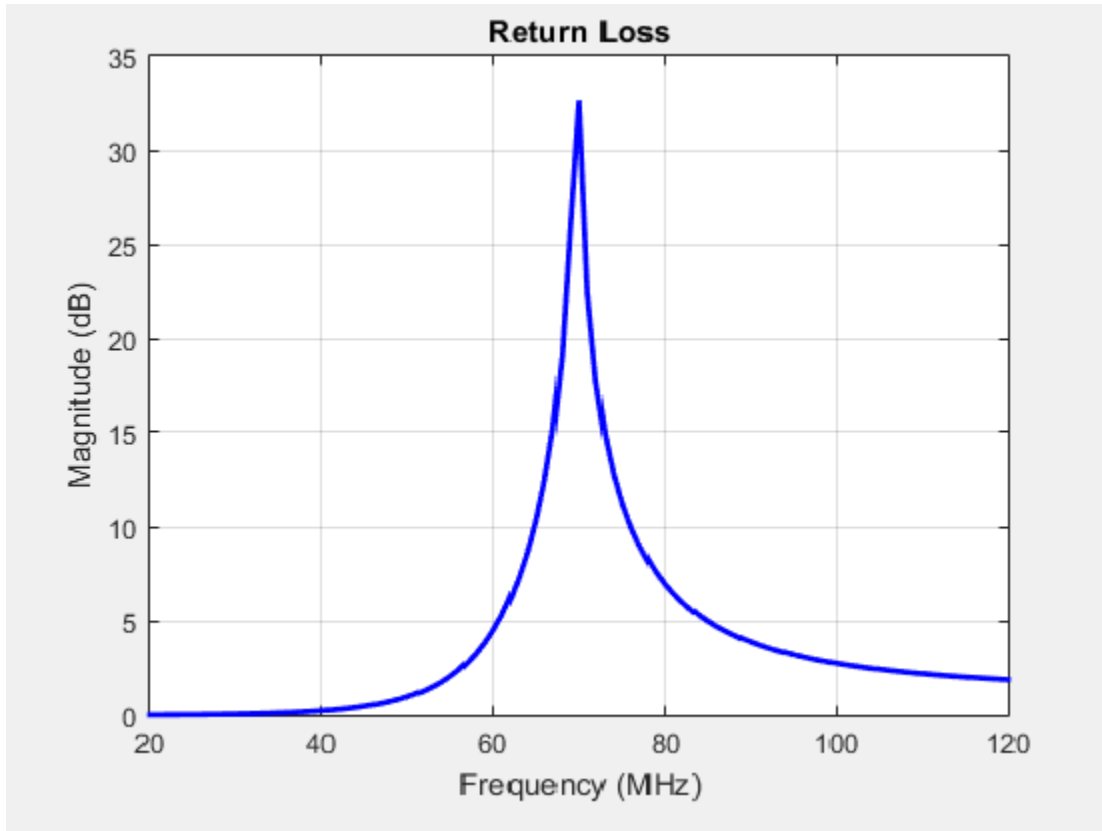
Return loss plots also give the operating bandwidth of the antenna. Antenna bandwidth is the frequency band over which the magnitude of return loss is greater than 10 dB. Use the returnLoss function to calculate the return loss of any antenna in the Antenna Toolbox library.

Voltage Standing Wave Ratio

The *voltage standing wave ratio* (VSWR) of an antenna is another measure of impedance matching between transmission line and antenna. The standing wave is generated because of the impedance mismatch at the port. VSWR is given by the equation:

$$VSWR = \frac{1+|S_{11}|}{1-|S_{11}|}$$

where:

- S_{11} is the reflection coefficient.

```
d=dipole;
vswr(d,20e6:1e6:120e6,72)
axis([20 120 120])
```
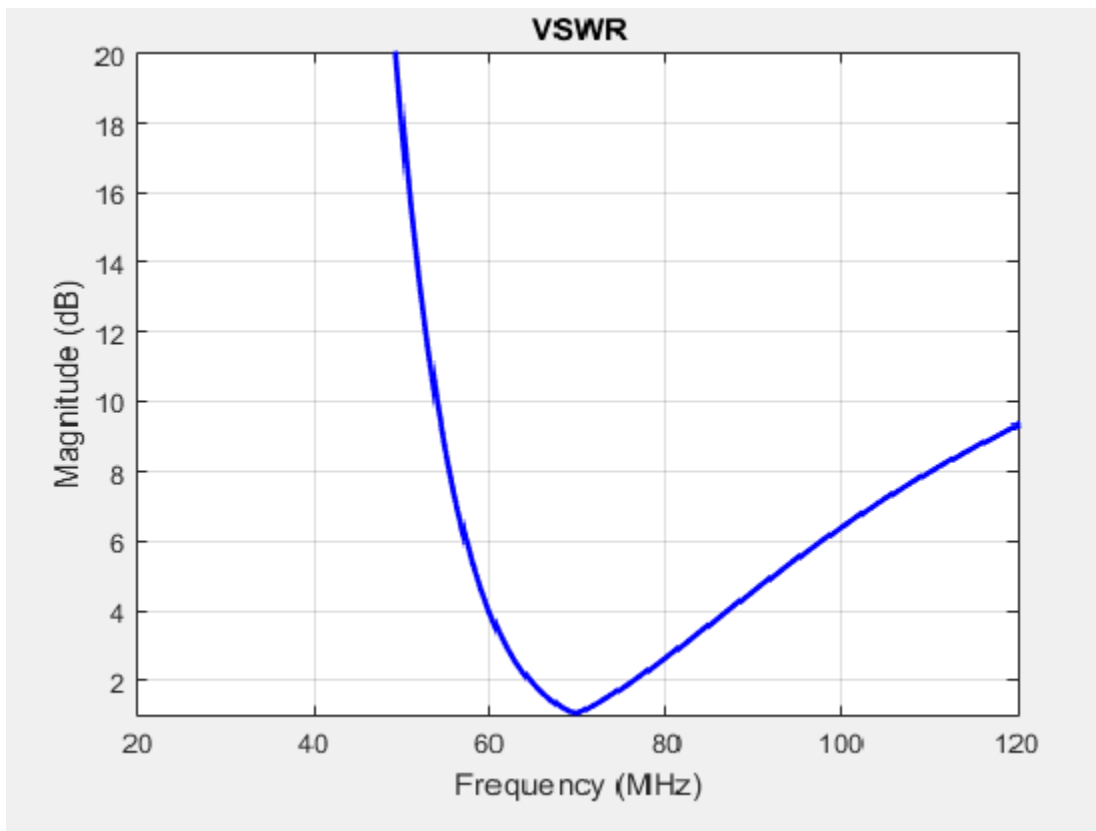

VSWR is scalar and contains no phase information. The value of VSWR lies between 1 and infinity. Antenna bandwidth is usually the frequency band over which the VSWR is less than approximately 2.

Use `vswr` to calculate the voltage standing wave ratio for any antenna in Antenna Toolbox.

Bandwidth

Bandwidth describes the range of frequencies over which the antenna can properly radiate or receive energy. It is a fundamental antenna parameter. Often, the desired bandwidth is one of the parameters used to determine which antenna to use. Antenna bandwidth is usually the frequency band over which the magnitude of the reflection coefficient is below -10 dB, or the magnitude of the return loss is greater than 10 dB, or the VSWR is less than approximately 2. All these criteria are equivalent. The bandwidth is controlled by the proper antenna design.

References

[1] Balanis, C.A. *Antenna Theory: Analysis and Design.*3rd Ed. New York: Wiley, 2005.

[2] Stutzman, Warren L., and Thiele, Gary A. *Antenna Theory and Design.* 3rd Ed. New York: Wiley, 2013.

[3] Bird, T.S. "Definition and Misuse of Return Loss." *IEEE Antennas and Propagation Magazine.* Vol. 51, Issue 2, April 2009, pp. 166–167.

Surface Analysis

In this section...
"Charge Distribution" on page 1-11
"Current Distribution" on page 1-15

Charge Distribution

The flow of charges on the antenna surface determines the surface currents of the antenna. For antennas to radiate, there must be acceleration or deceleration of charges. The deceleration of charges is caused due to buildup of charges at the end of the wire, which leads to impedance discontinuities. This mechanism creates electromagnetic radiation. The accumulation of charges varies according to time and structure of the antenna.

The accumulation of charges is exploited in many ways. If you calculate the impedance of this monopole antenna using the impedance function, you get the following plot:

```
m = monopole
impedance(m,20e6:1e6:120e6)
```
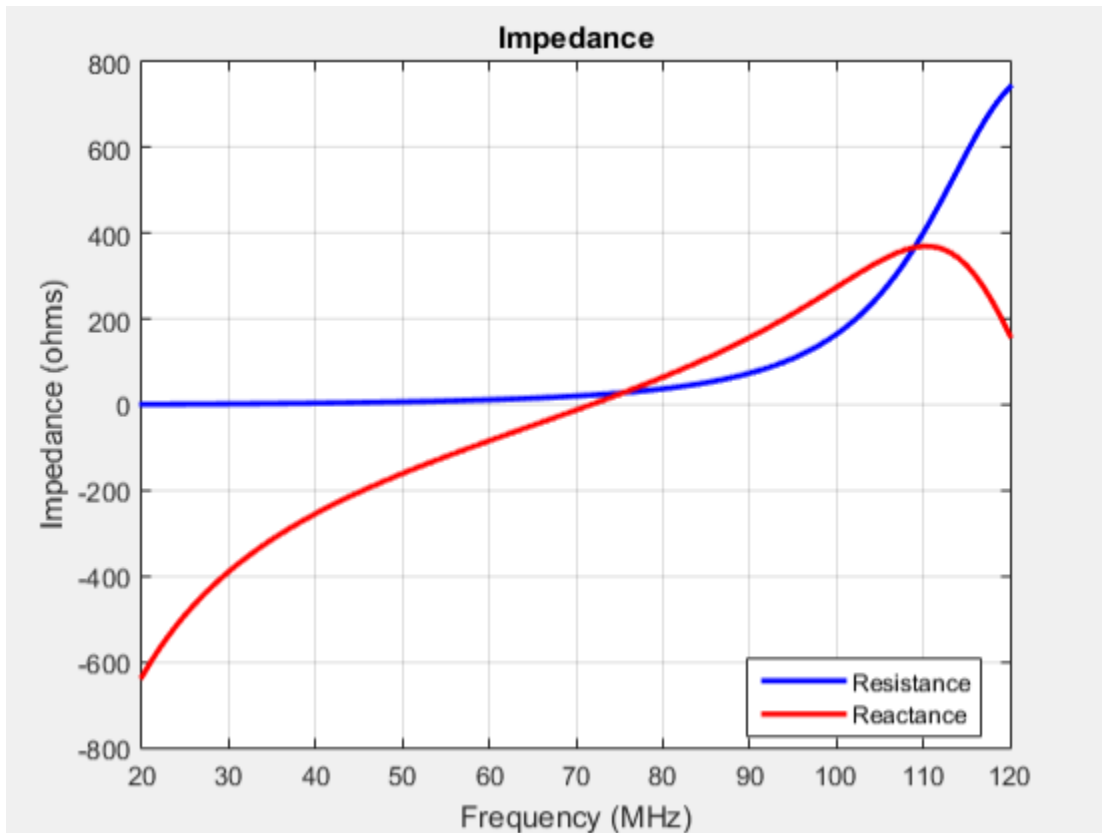
You can observe the first resonance is at approximately 71 MHz. To lower the resonance frequency, recalculate the height of the monopole to quarter wavelength. The frequency of operation is also lower. You must also have to increase the size of the corresponding ground plane. This increase in size means that to achieve similar performance at a lower frequency, you need a larger antenna. This approach might not be possible due to physical space constraints.

Alternatively, you can exploit the fact that antennas have charge accumulation. If you provide appropriate structural modification to the antenna, charges accumulate. For a monopole antenna, you can enable charge accumulation by adding a top-hat to the monopole. Now, if you calculate the impedance of the antenna using the top-hat, the plot is:

```
mt = monopoleTopHat
```

```
impedance(mt,20e6:1e6:120e6)
```
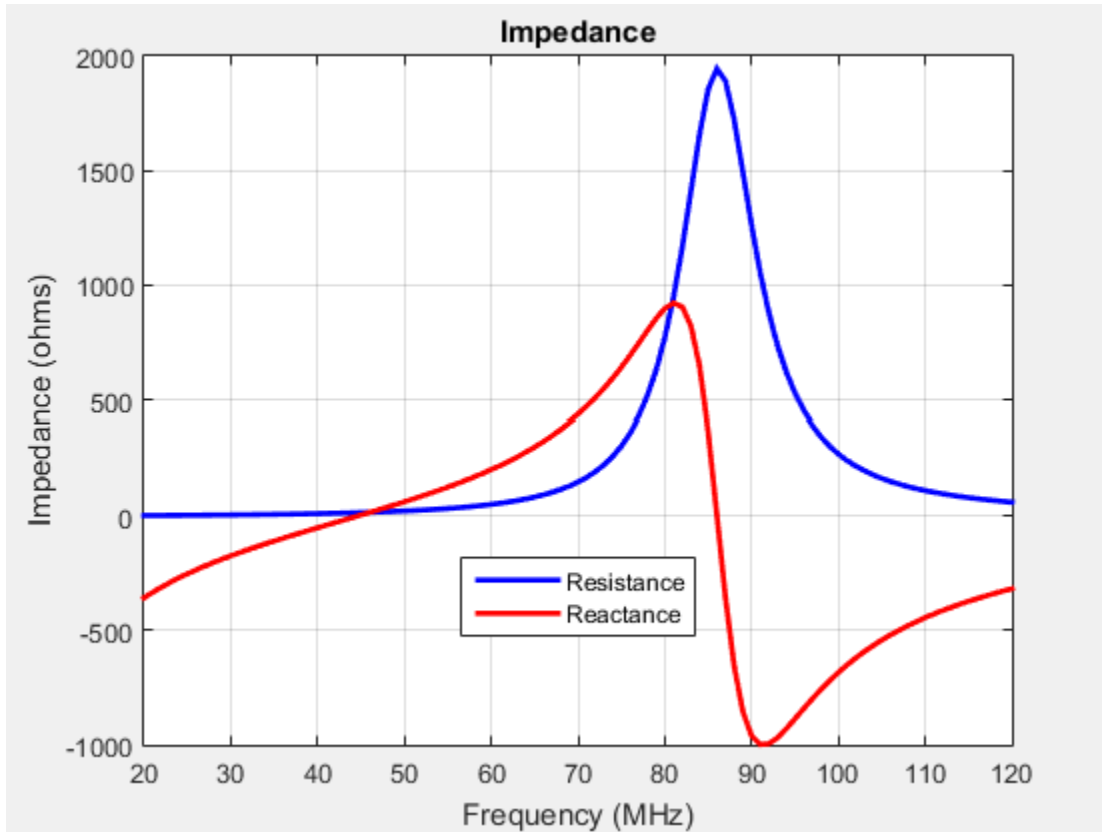

All the dimensions of this antenna are same as the monopole. The first resonance of the antenna is approximately 45 MHz. To view the accumulated charges on the top-hat monopole, use the charge function:
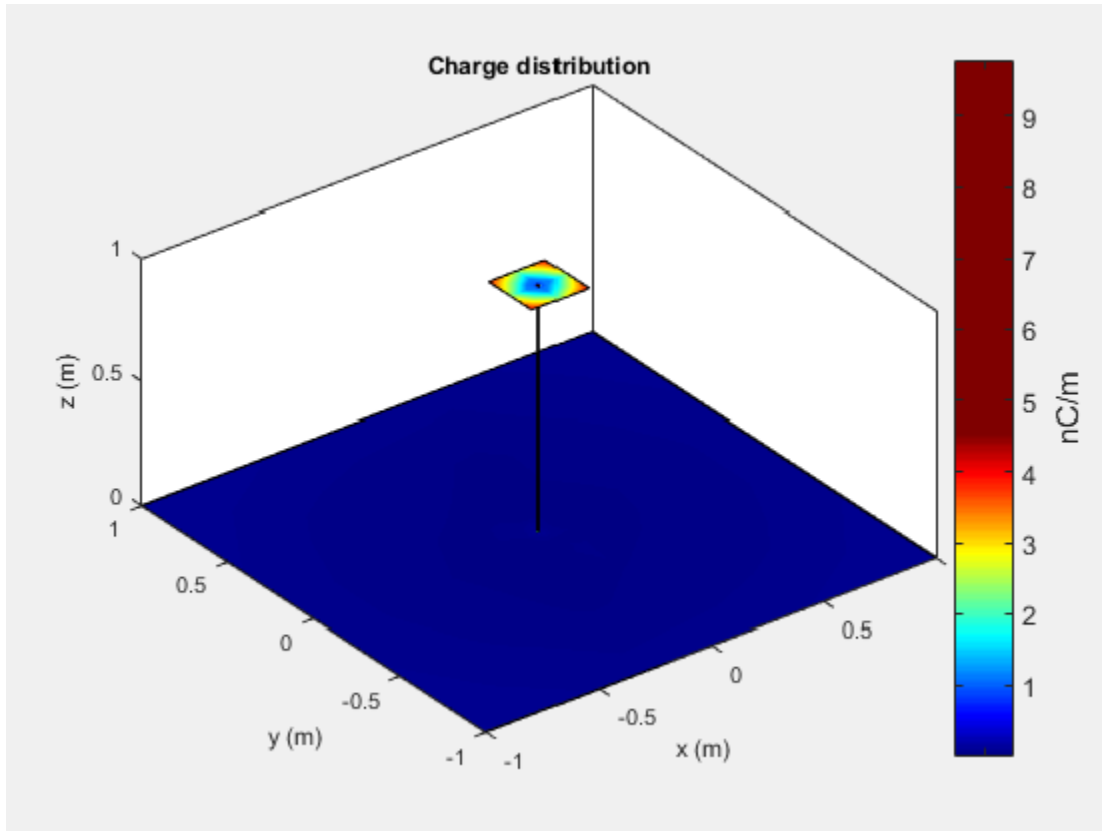
The increase in capacitance lowers the frequency of the antenna. By keeping the physical volume of the antenna same, the resonance point is shifted.

Increasing the top-hat dimensions provides more surface area for charges to accumulate thereby increasing the capacitance, which in turn pushes the resonant frequency lower. For example:

```
mt.TopHatLength = 0.35
mt.TopHatWidth = 0.35
impedance(mt,20e6:1e6:120e6)
```
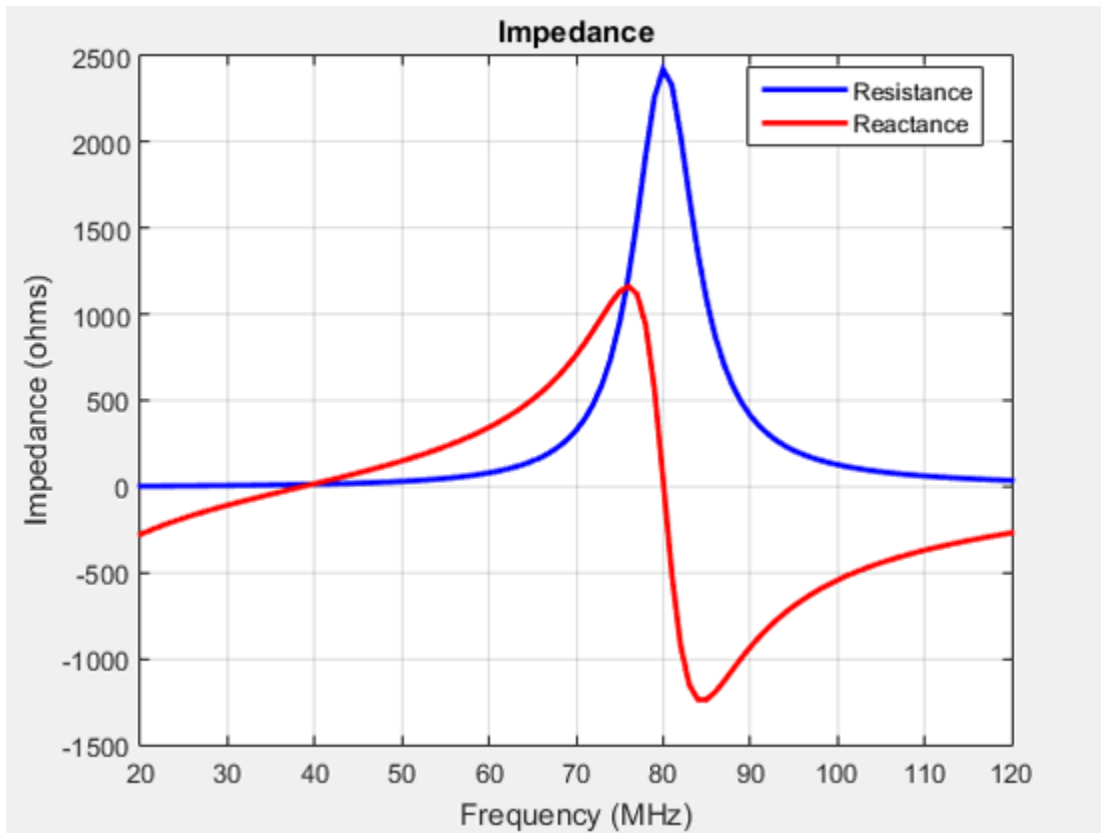
The resonance of the antenna further reduces to approximately 40 MHz.

Current Distribution

A typical antenna surface has current flowing on it. The behavior of the antenna surface current depends on the frequency of the input source, the geometry of the antenna, and the material properties of the antenna. The current is a vector and is spatially related to the structure of the antenna. In a dipole antenna, the maximum current distribution in the middle of the antenna and the minimum is toward the end:

```
d = dipole;
current(d,70e6)
```

Current distribution
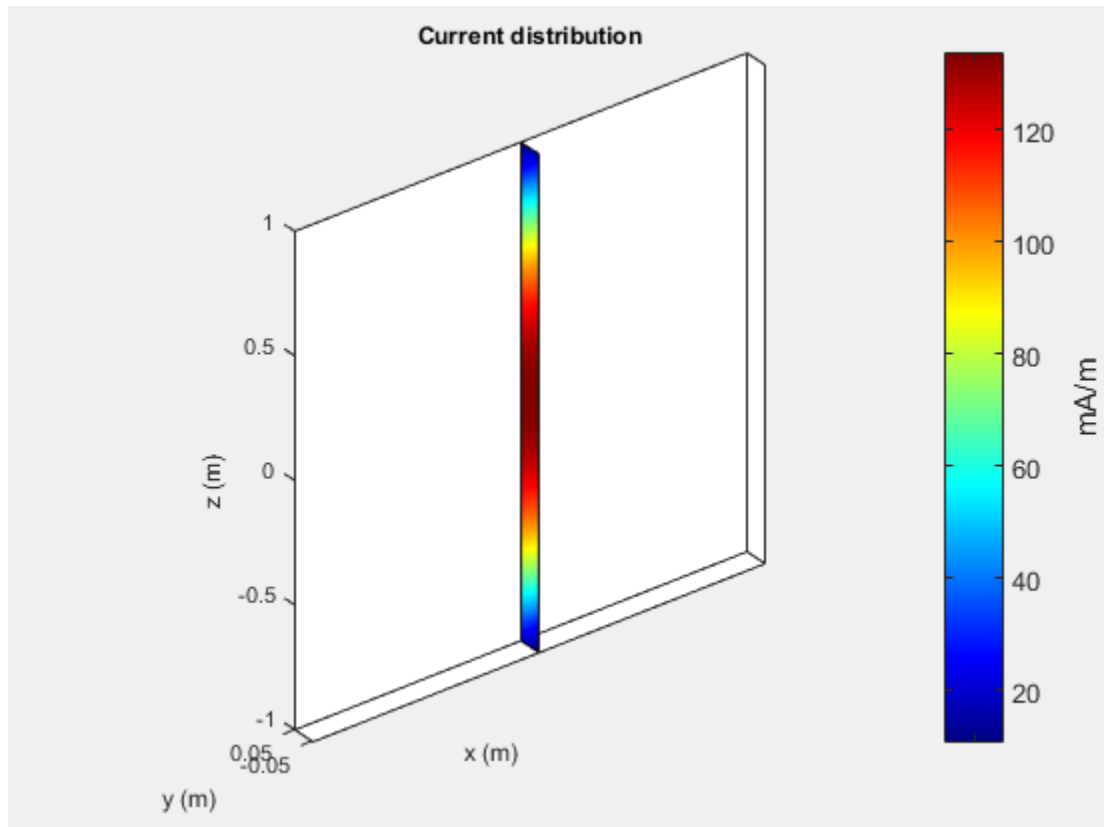
The same is true for a spiral antenna:

```
s = spiralEquiangular;
current(s,4e9)
```
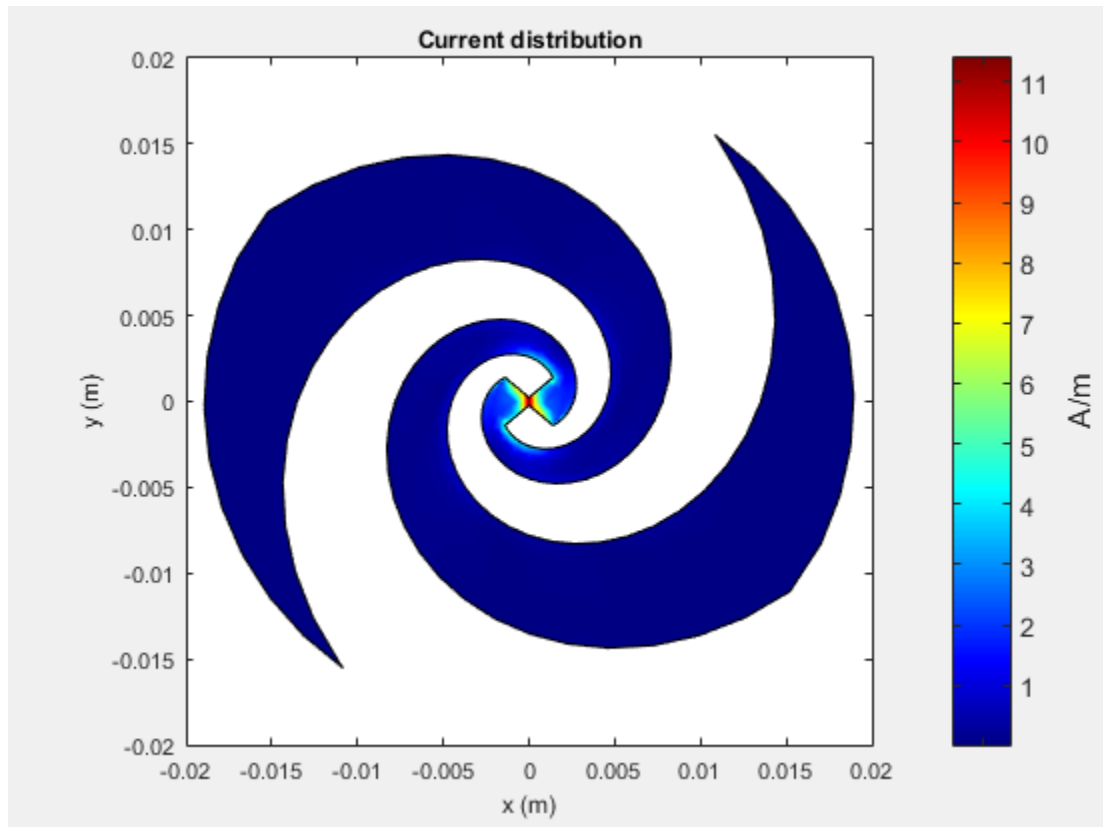
The patch also shows the current distribution of the classic $\lambda/2$ open-open resistor. The two ends of the patch represent an open circuit since the current is at a minimum.

```
pm = patchMicrostrip;
current(pm,1.75e9)
```
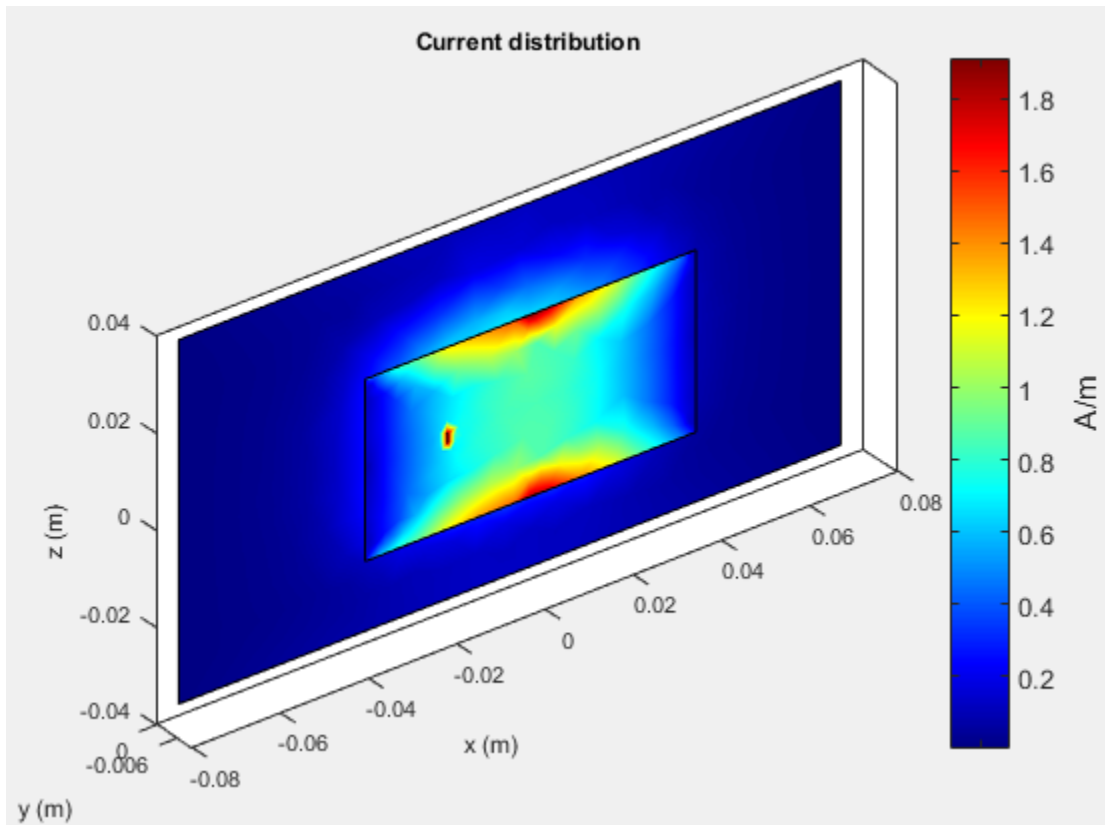
The spatial relationship between the current and structure of antenna is termed mode.

References

[1] Balanis, C.A. *Antenna Theory. Analysis and Design*, 3rd Ed. New York: Wiley, 2005.

[2] Makarov, S.N. *Antenna and EM Modeling with MATLAB*, New York: Wiley & Sons, 2002, p. 66.

Meshing

Method of Moments (MoM) is a numerical method that transforms Maxwell's the continuous integral equations into an approximate discrete formulation that requires inversion of a large matrix. *Meshing* is the process of converting the continuous domain into the discrete domain for solving the equations. For discretizing surfaces, typically either triangles or rectangles are used. Antenna Toolbox uses triangular element for meshing as it conforms better for arbitrary shaped surfaces. The triangles are used to approximate the surface current using the Rao-Wilton-Glisson (RWG) basis functions. To get an accurate result, ensure that large number of triangles are present in the region where current variation is the highest. This region is typically either the corners in the antenna geometry or at the point where the antenna is excited.

In this section...
"Strip Meshing" on page 1-19
"Surface Meshing" on page 1-20
"Automatic Meshing Mode" on page 1-21
"Manual Meshing Mode" on page 1-22

Strip Meshing

For strip meshing, include at least 10 triangles per wavelength in a strip. This rule applies for structures such as dipoles, monopoles, and loops. Antenna Toolbox antennas meets the requirement automatically, based on the analysis frequency specified. The structured mesh generated in such cases is shown:

NumTriangles : 160
MaxEdgeLength : 0.025
MeshMode : manual
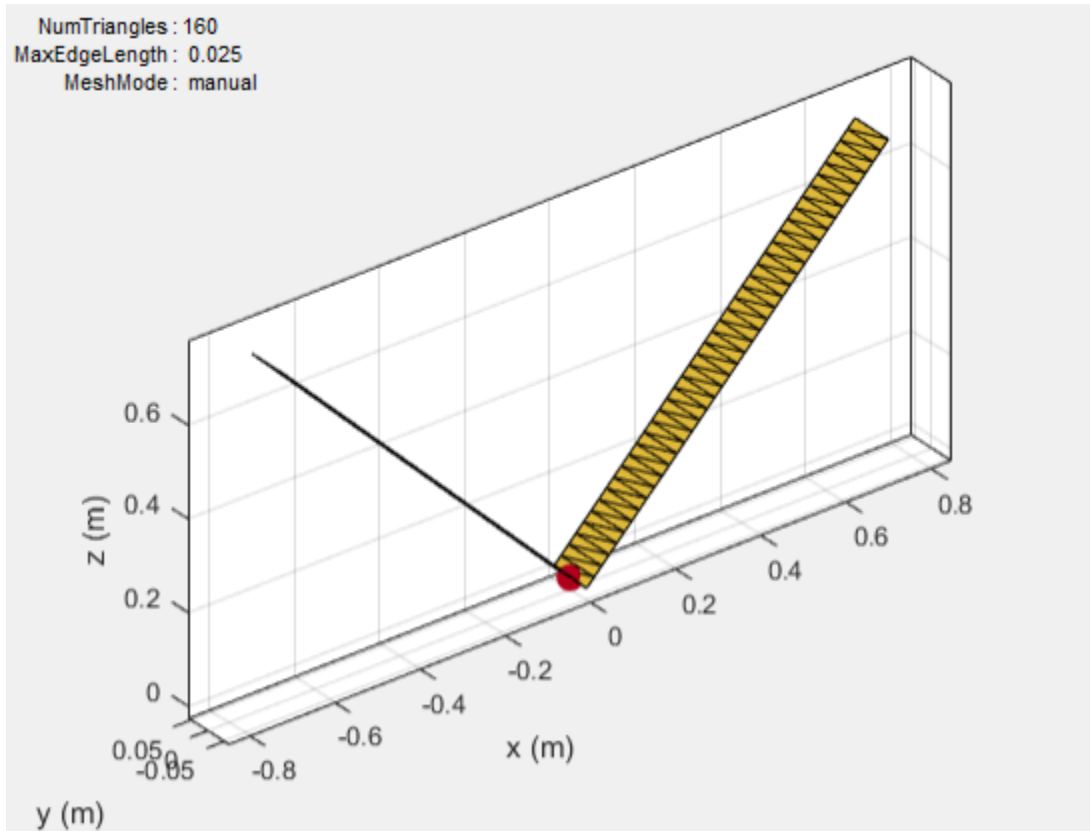
Surface Meshing

For surface meshing, it is recommended that there be at least 100 elements per wavelength in a particular area. This rule applies to structures such as spirals, patches, and ground planes in general. Antenna Toolbox antennas meets the requirement automatically, based on the analysis frequency specified. In these cases, a non-uniform mesh is generated as shown:

NumTriangles : 854
MaxEdgeLength : 0.025
MeshMode : manual
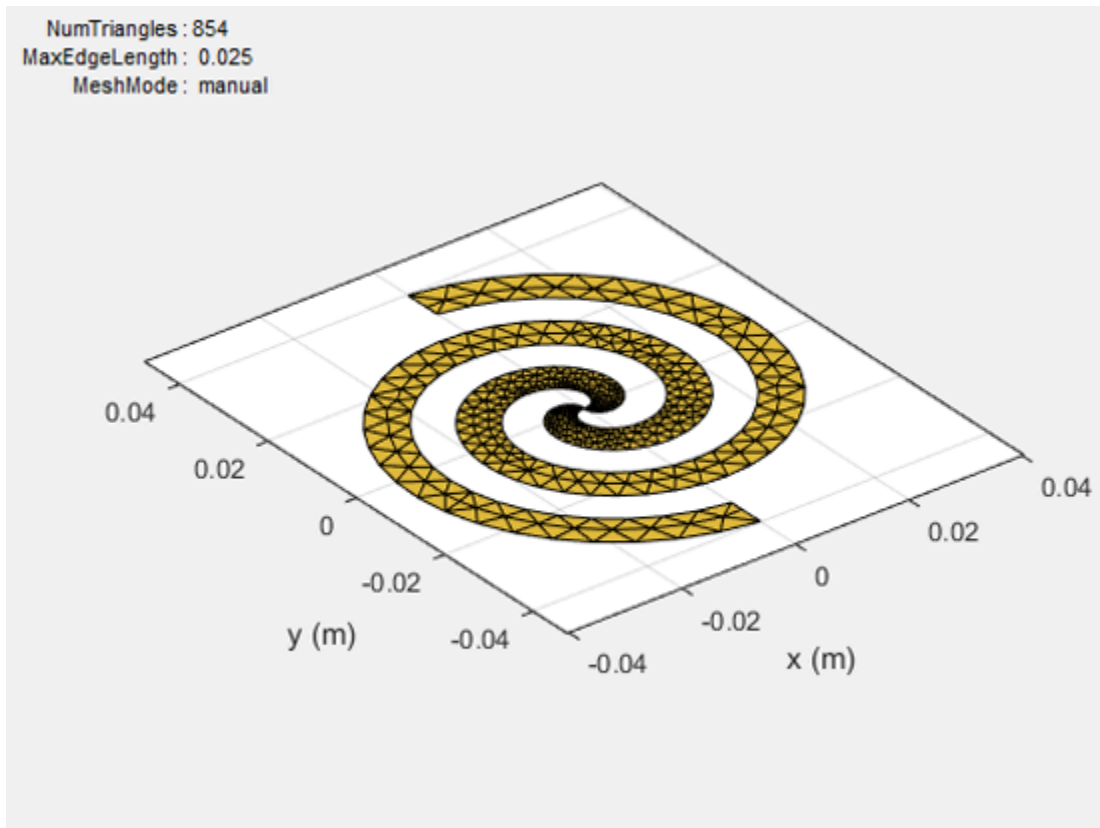
Larger number of triangles are added in regions with higher current density.

Automatic Meshing Mode

As mentioned in Strip Meshing and Surface Meshing, the antenna structures mesh automatically based on the analysis frequency chosen. For analysis functions that accept a scalar frequency, the antennas mesh at every frequency to satisfy the minimum triangles required. Then the functions calculate the corresponding antenna parameters. For analysis functions that accept a frequency vector (`impedance`, `sparameters`, `returnLoss`, `vswr`), each antenna meshes once at the highest frequency. Then, the functions calculate corresponding antenna parameters at all the frequencies in the range.

Manual Meshing Mode

Automatic meshing at every frequency can increase the computation time drastically. To get around this delay, you can choose to mesh the structure manually at the highest frequency of interest. Manual meshing is done by specifying the maximum edge length that is used for discretizing the structure. One option is to specify the value to be one-tenth of the wavelength at the highest frequency of interest. For example:

```
sp    = spiralArchimedean;
freq  = 0.8e9:100e6:2.5e9;
lambda = 3e8/freq(end);
mesh (sp,'MaxEdgeLength',lambda/10);
```

Alternatively, you can run an analysis at the highest frequency of interest and get the maximum edge length. Specify this maximum edge length using the `mesh` function as shown. This mesh is used for all other calculations.

```
sp    = spiralArchimedean;
freq  = 0.8e9:100e6:2.5e9;
temp  = axialRatio(sp,freq(end), 0, 90);
meshdata = mesh(sp);
mesh(sp,'MaxEdgeLength',meshdata.MaxEdgeLength);
```

References

[1] Makarov, S.N. *Antenna and EM Modeling with MATLAB*, New York: Wiley & Sons, 2002

Field Analysis

Radiation Pattern

The *radiation pattern* of an antenna is the spatial distribution of power. The pattern displays the directivity or gain of the antenna. the *power pattern* of an antenna plots the transmitted or received power for a given radius. The *field pattern* of an antenna plots the variation in the electric or magnetic field for a given radius. The radiation pattern provides details such as the maximum and minimum value of the field quantity and the range of angles over which data is plotted.

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
pattern(h,2.1e9)
```

Output : Directivity
Frequency : 2.1 GHz
Max value : 14.1 dBi
Min value : -12.7 dBi
Azimuth : [-180° , 180°]
Elevation : [-90° , 90°]

Main Lobe

Side Lobe

Null

Back Lobe
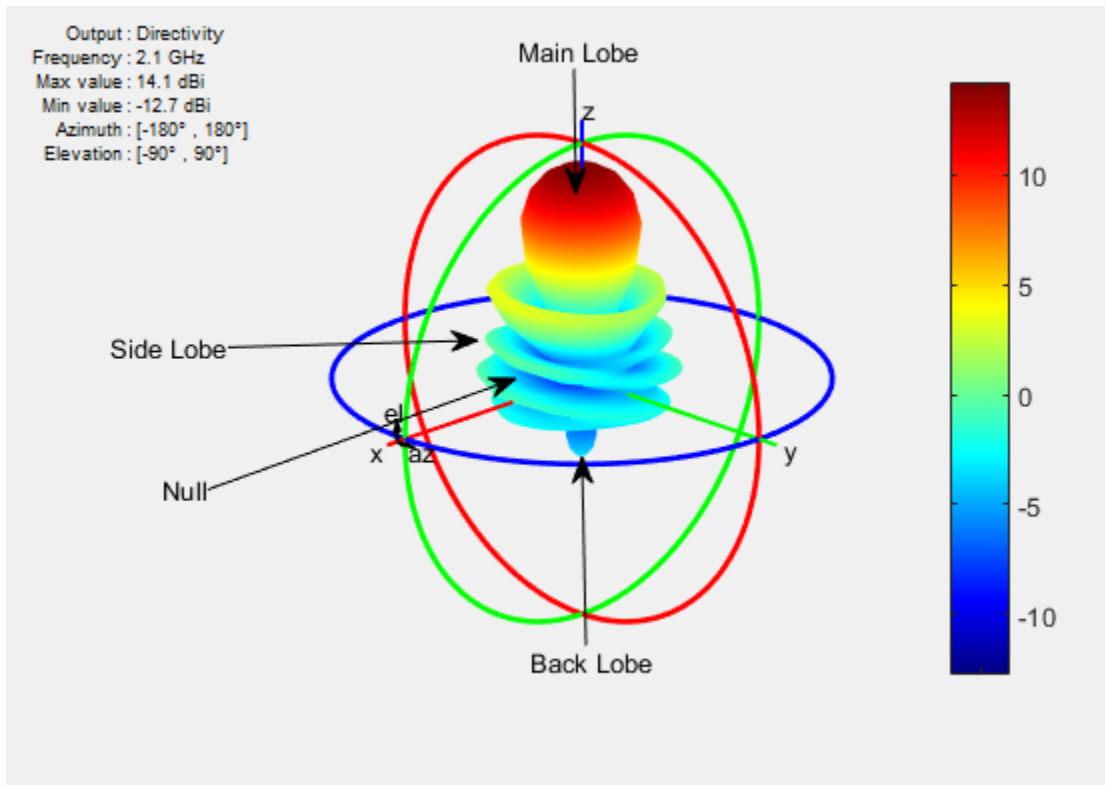
Use the `pattern` function to plot radiation pattern of any antenna in the Antenna Toolbox. By default, the function plots the directivity of the antenna. You can also plot the electric field and power pattern by using *Type* name-value pair argument of the pattern function.

Lobes

Each radiation pattern of an antenna contains *radiation lobes*. The lobes are divided into *major lobes* (also called main lobes) and *minor lobes*. *Side lobes* and *back lobes* are variations of minor lobes.

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
patternElevation(h,2.1e9)
```

Output : Directivity
Frequency : 2.1 GHz
Max value : 14.1 dBi
Min value : -11.7 dBi
Azimuth : 0°
Elevation : [1° , 360°]
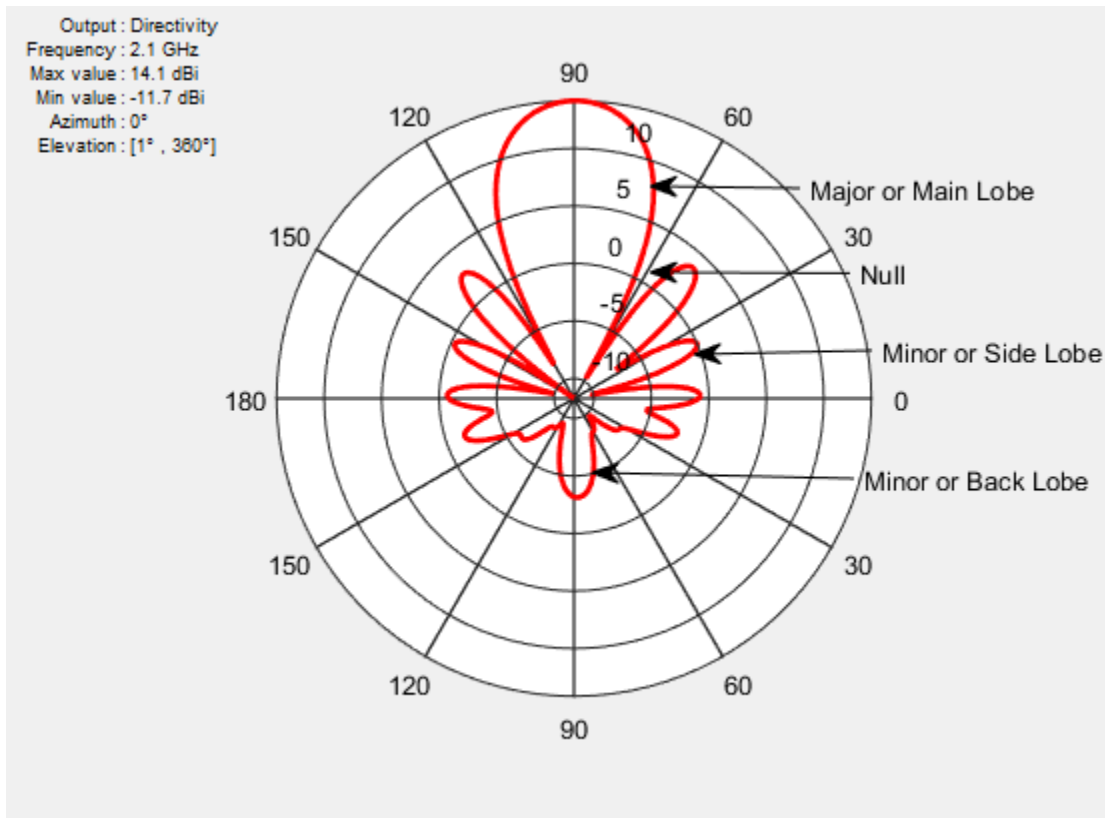
- `Major or Main lobe`: Shows the direction of maximum radiation, or power, of the antenna.
- `Minor lobe`: Shows the radiation in undesired directions of antenna. The fewer the number of minor lobes, the greater the efficiency of the antenna. *Side lobes* are minor lobes that lie adjacent to the major lobe. *Back lobes* are minor lobes that lie opposite to the major lobe of antenna.
- `Null`: Shows the direction of zero radiation intensity of the antenna. Nulls usually lie between the major and minor lobe or in between the minor lobes of the antennas.

Field Regions

For an antenna engineer and an electromagnetic compatibility (EMC) engineer, it is important to understand the regions around the antenna.
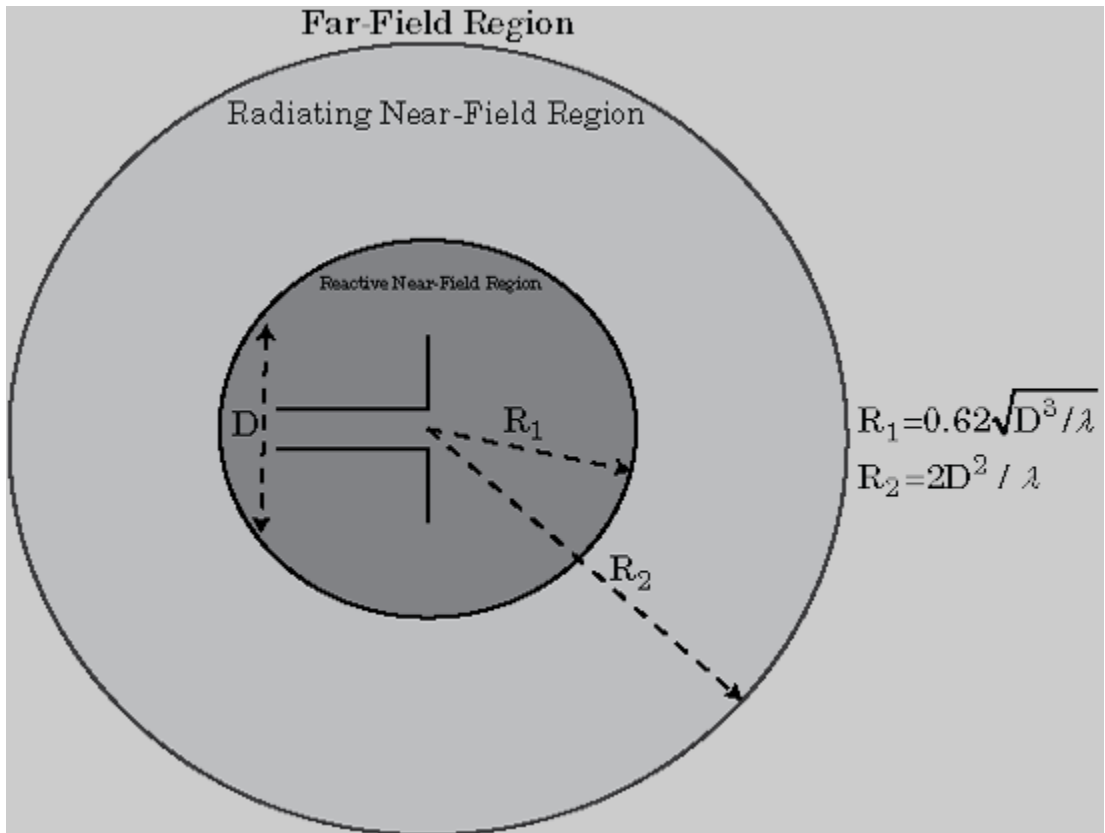
The region around an antenna is defined in many ways. The most used description is a 2- or 3-region model. The 2-region model uses the terms *near field* and the *far field* to identify specific dominant field mechanisms. The diagram is a representation of antenna fields and boundaries. The 3-field region splits the near field into a transition zone, where a weakly radiative mechanism is at work.

`Near-Field Region`: The *near-field region* is divided into two transition zones: a reactive zone and radiating zone.

- `Reactive Near-Field Region`: This region is closest to the antenna surface and is dominated by the reactive field. The reactive field is stored energy, or standing waves. The fields in this region change rapidly with distance from the antenna. The

   outer boundary of this region is given by the equation $R < 0.62\sqrt{D^3 / \lambda}$ where R is the

distance from the antenna, λ is the wavelength and D is the largest dimension of the antenna. This equation holds true for most antennas. In a very short dipole, the outer boundary of this region is given by $\lambda / 2\pi$ from the antenna surface.

- `Radiating Near-Field Region`: This region is also called the *Fresnel region* and lies between the reactive near-field region and the far-field region. The existence of this region depends on the largest dimension of the antenna and the wavelength of operation. The radiating fields are dominant in this region. The equation for the inner boundary of the region is equation $R \geq 0.62\sqrt{D^3 / \lambda}$ and the outer boundary is $R < 2D^2 / \lambda$. This holds true for most antennas. The filed distribution depends on the distance from the antenna.

`Far-field Region`: This region is also called *Fraunhofer region*. In this region, the field distribution does not depend on the distance from the antenna. The electric and magnetic fields in this region are orthogonal to each other. This region contains propagating waves. The equation for the inner boundary of the far-field is $R = 2D^2 / \lambda$ and the equation for the outer boundary is infinity.

Directivity and Gain

Directivity is the ability of an antenna to radiate power in a particular direction. It can be defined as ratio of maximum radiation intensity in the desired direction to the average radiation intensity in all other directions. The equation for directivity is:

$$D = \frac{4\pi U(\theta, \phi)}{P_{rad}}$$

where:

- D is the directivity of the antenna
- U is the radiation intensity of the antenna
- P_{rad} is the average radiated power of antenna in all other directions

Antenna directivity is dimensionless and is calculated in decibels compared to the isotropic radiator (dBi)..

The *gain* of an antenna depends on the directivity and efficiency of the antenna. It can be defined as the ratio of maximum radiation intensity in the desired direction to the total power input of the antenna. The equation for gain of an antenna is:

$$G = \frac{4\pi U(\theta,\phi)}{P_{in}}$$

where:

- G is the gain of the antenna
- U is the radiation intensity of the antenna
- P_{in} is the total power input to the antenna

If the efficiency of the antenna in the desired direction is 100%, then the total power input to the antenna is equal to the total power radiated by the antenna, that is, $P_{in} = P_{rad}$. In this case, the antenna directivity is equal to the antenna gain.

Beamwidth

Antenna beamwidth is the angular measure of the antenna pattern coverage. As seen in the figure, the main beam is a region around maximum radiation. This beam is also called the major lobe, or main lobe of the antenna.
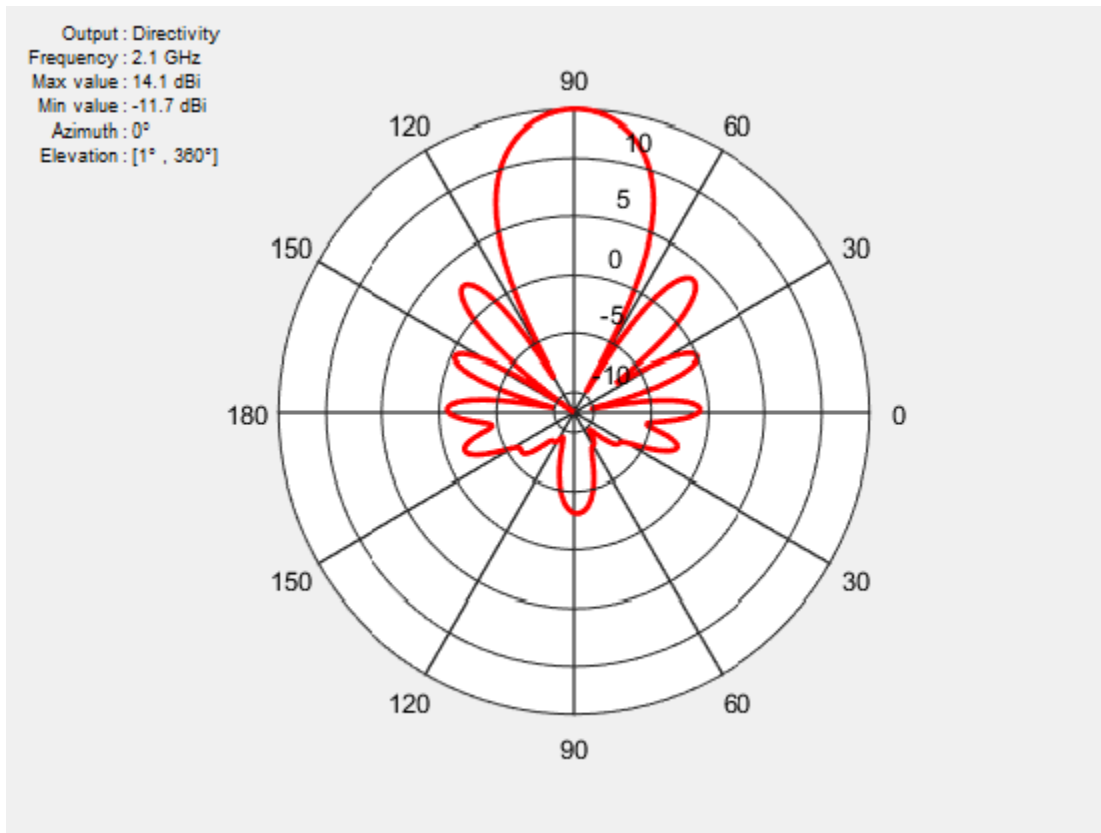
Half power beamwidth (HPBW) is the angular separation in which the magnitude of the radiation pattern decreases by 50% (or -3dB) from the peak of the main beam

Use the beamwidth function to calculate the beamwidth of any antenna in Antenna Toolbox.

E-Plane and H Plane

E-plane: Plane containing the electric field vector and the direction of maximum radiation. Consider a dipole antenna that is vertical along the z-axis. Use the patternElevation function to plot the elevation plane pattern. The elevation plane pattern shown captures the E-plane behavior of the dipole antenna.

```
d=dipole;
patternElevation(h,70e6)
```
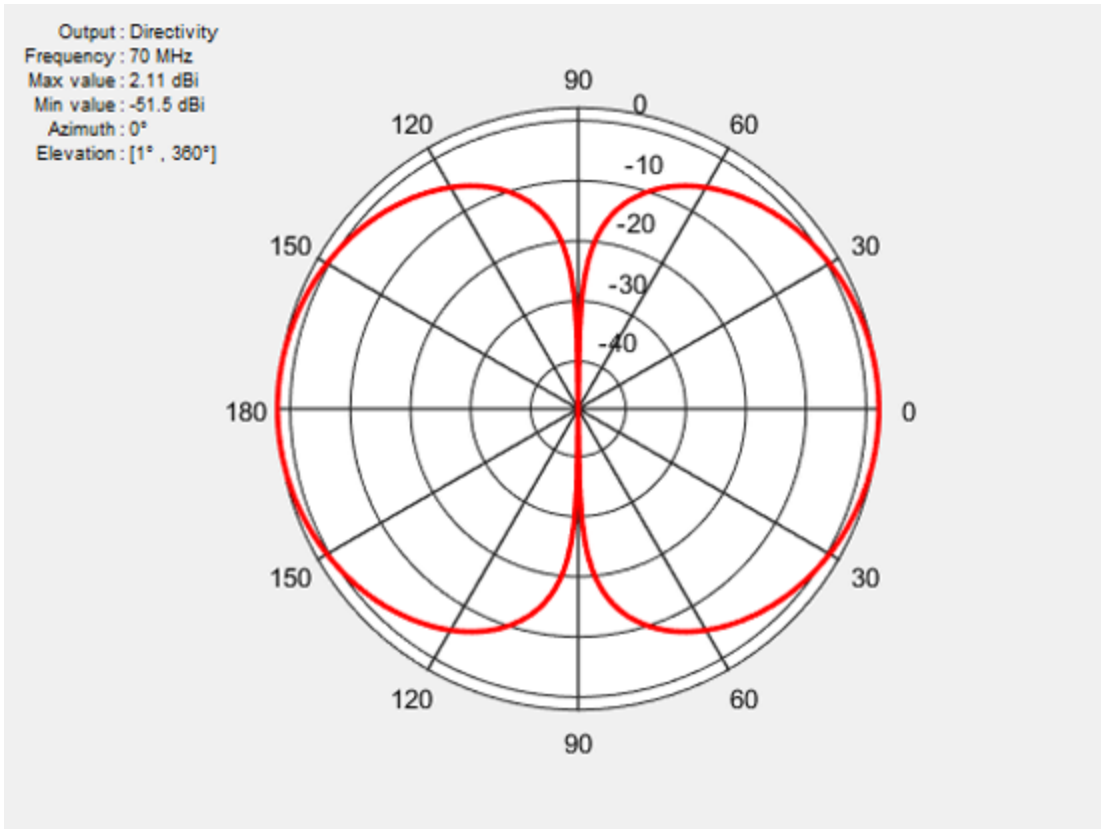

H-plane: Plane containing the magnetic field vector and the direction of maximum radiation. Use the `patternAzimuth` function to plot the azimuth plane pattern of a dipole antenna. The azimuthal variation in pattern shown captures the H-plane behavior of the dipole antenna.

```
d=dipole;
patternAzimuth(h,70e6)
```
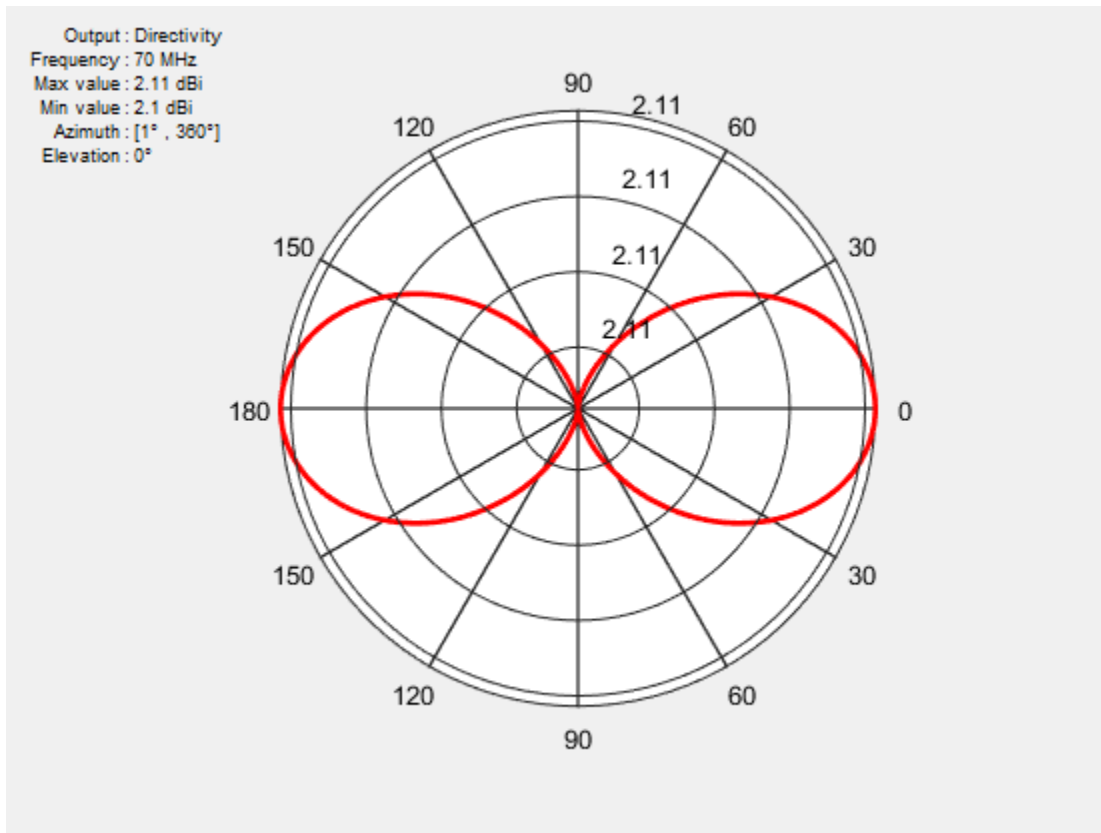
Use EHfields to measure the electric and magnetic fields of the antenna. The function can be used to calculate both near and far fields.

Polarization

Polarization is the orientation of the electric field, or E-field, of an antenna. Polarization is classified as elliptical, linear, or circular.

Elliptical polarization: If the electric field remains constant along the length but traces an ellipse as it moves forward, the field is elliptically polarized. Linear and circular polarizations are special cases of elliptical polarization.

Linear polarization: If the electric field vector at a point in space is directed along a straight line, the field is linearly polarized. A linearly polarized antenna radiates only one plane and this plane contains the direction of propagation of the radio waves. There are two types of linear polarization:

- Horizontal Polarization: The electric field vector is parallel to the ground plane. To view the horizontal polarization pattern of an antenna, use the pattern function, with the 'Polarization' name-value pair argument set to 'H'. The plot shows the horizontal polarization pattern of a dipole antenna:

```
d=dipole;
pattern(d,70e6,'Polarization','H')
```
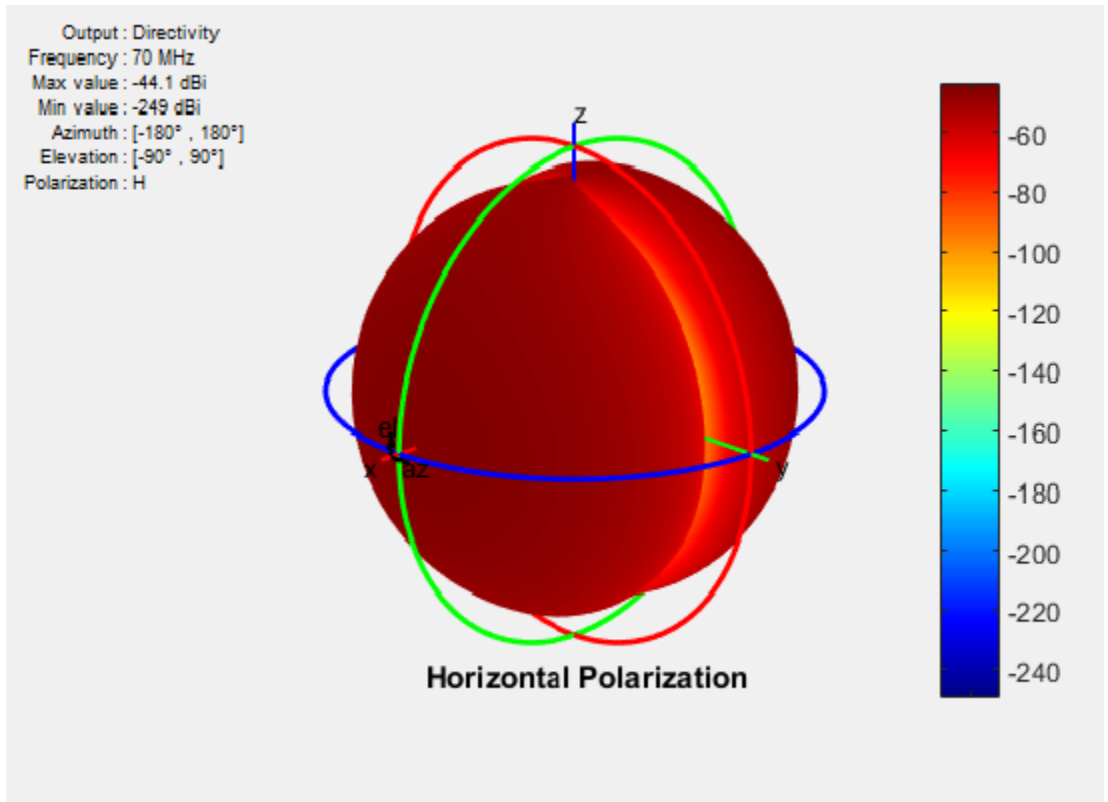

USA television networks use horizontally polarized antennas for broadcasting.

- `Vertical Polarization`: The electric field vector is perpendicular to the ground plane. To view the vertical polarization pattern of an antenna, use the `pattern` function, with the 'Polarization' name-value pair argument set to 'V'. Vertical polarization is used when a signal has to be radiated in all directions. The plot shows the vertical polarization pattern of a dipole antenna:

```
d=dipole;
pattern(d,70e6,'Polarization','V')
```
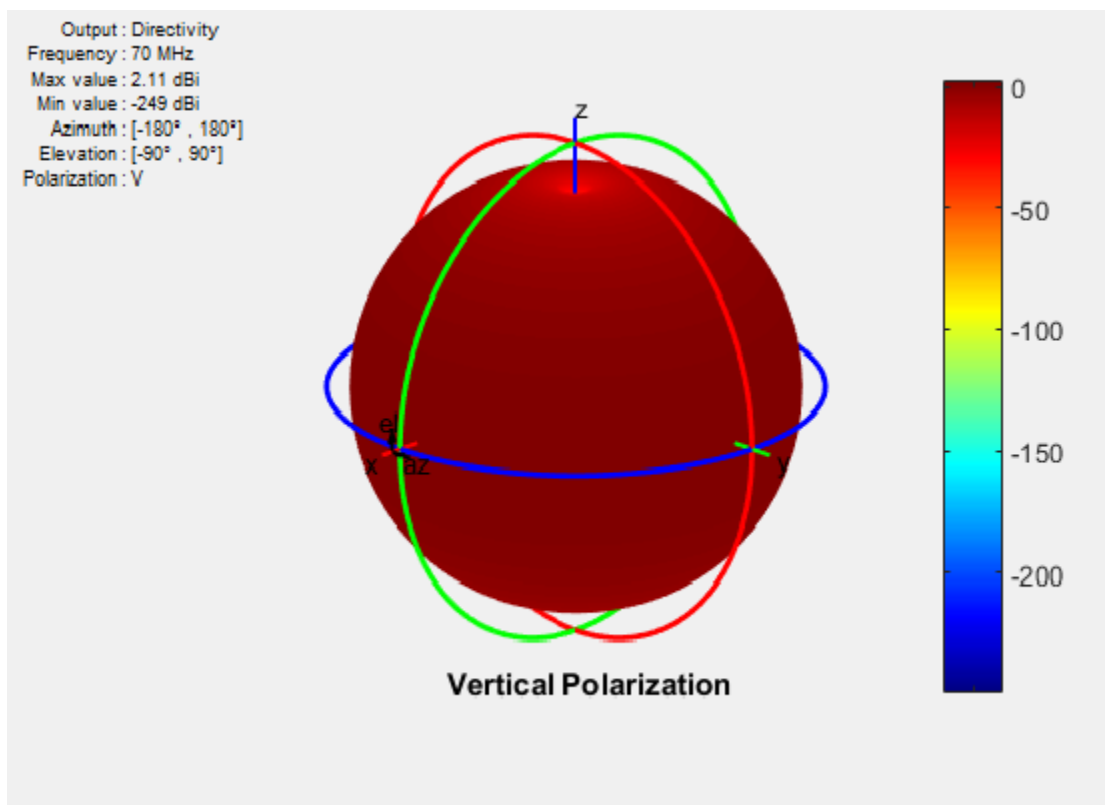

An AM radio broadcast antenna or an automobile whip antenna are some examples of vertically polarized antennas.

`Circular Polarization`: If the electric field remains constant along the straight line but traces circle as it moves forward, the field is circularly polarized. This wave radiates

in both vertical and horizontal planes. Circular polarization is most often used in satellite communications. There are two types of circular polarization:

- `Right-Hand Circularly Polarized (RHCP)`: The electric field vector is traced in the counterclockwise direction. To view the RHCP pattern of an antenna, use the `pattern` function, with the 'Polarization' name-value pair argument set to 'RHCP'. The plot shows RHCP pattern of helix antenna:

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
pattern(h,1.8e9,'Polarization','RHCP')
```


Output : Directivity
Frequency : 1.8 GHz
Max value : 12.2 dBi
Min value : -46.4 dBi
Azimuth : [-180° , 180°]
Elevation : [-90° , 90°]
Polarization : RHCP

Right-Hand Circular Polarization
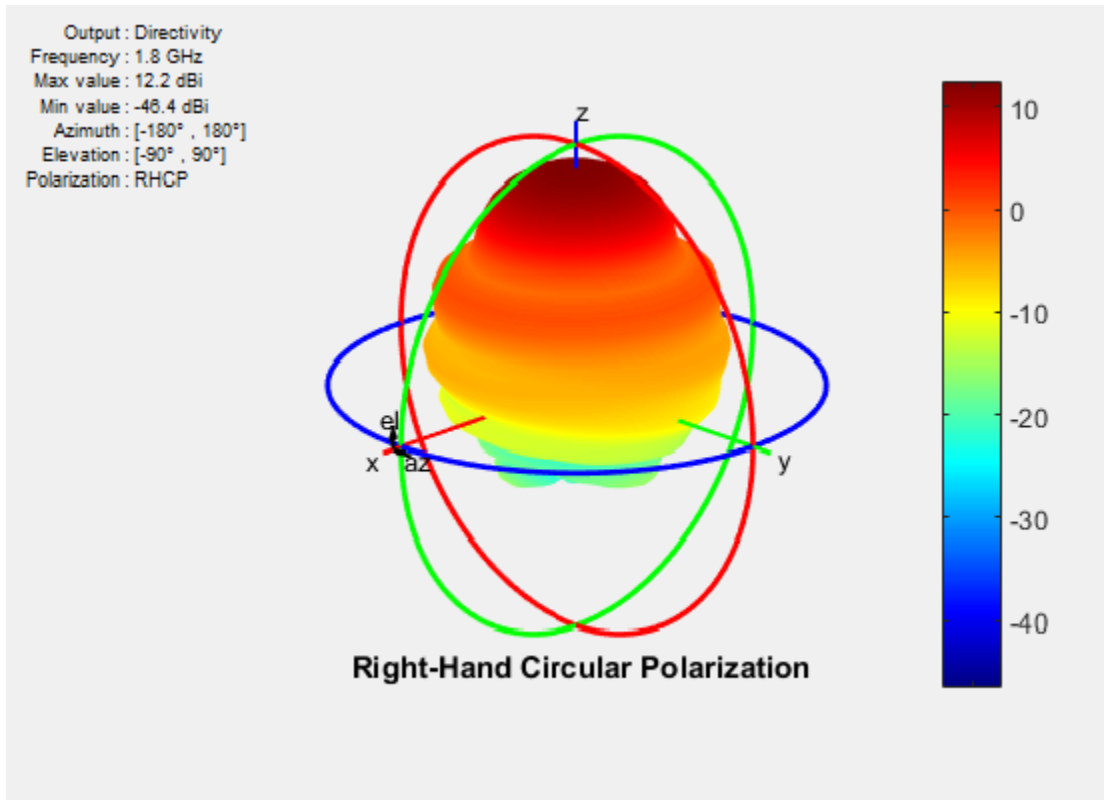
- `Left-Hand circularly polarized (LHCP)`: The electric field vector is traced in the clockwise direction. To view the LHCP pattern of an antenna, use the `pattern`

function, with the 'Polarization' name-value pair argument set to 'LHCP'. The plot shows LHCP pattern of helix antenna:

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
pattern(h,1.8e9,'Polarization','LHCP')
```
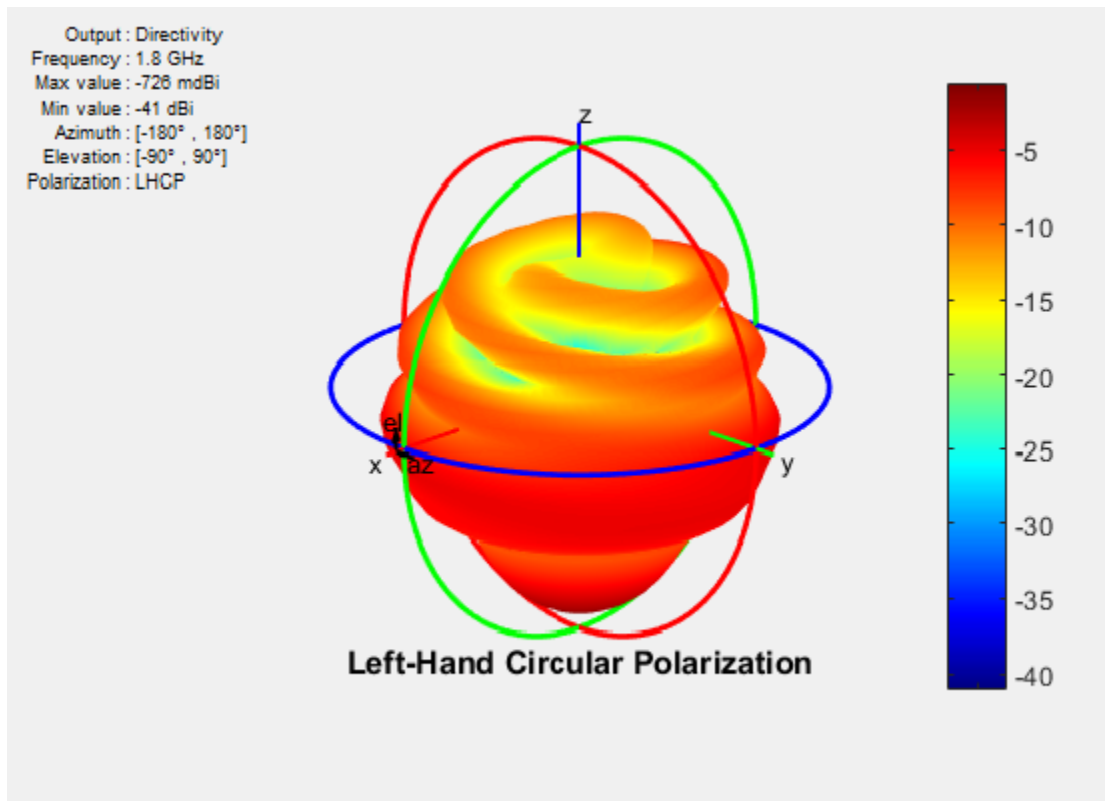

For efficient communications, the antennas at the transmitting and receiving end must have same polarization.

Axial Ratio

Axial ratio (AR) of an antenna in a given direction quantifies the ratio of orthogonal field components radiated in a circularly polarized wave. An axial ratio of infinity implies a linearly polarized wave. When the axial ratio is 1, the radiated wave has pure circular polarization. Values greater than 1 imply elliptically polarized waves.

Use `axialRatio` to calculate the axial ratio for any antenna in the Antenna Toolbox.

References

[1] Balanis, C.A. *Antenna Theory. Analysis and Design*, 3rd Ed. New York: Wiley, 2005.

[2] Stutzman, Warren.L and Thiele, Gary A. *Antenna Theory and Design*, 3rd Ed. New York: Wiley, 2013.

[3] Capps, C. *Near Field or Far Field*, EDN, August 16, 2001, pp.95 - pp.102.

Model Infinite Ground Plane

This example shows how to model an infinite ground plane and calculate fundamental antenna parameters.

Create Antenna On Infinite Ground Plane

Create a reflector antenna on a ground plane of infinite length.

```
r = reflector ('GroundPlaneLength', inf)


r =

  reflector with properties:

               Exciter: [1x1 dipole]
     GroundPlaneLength: Inf
      GroundPlaneWidth: 0.2000
               Spacing: 0.0750
                  Tilt: 0
              TiltAxis: [1 0 0]
```

View Antenna Geometry

View the physical construction of infinite ground plane reflector antenna.

```
figure;show(r);
```

dipole over infinite ground plane
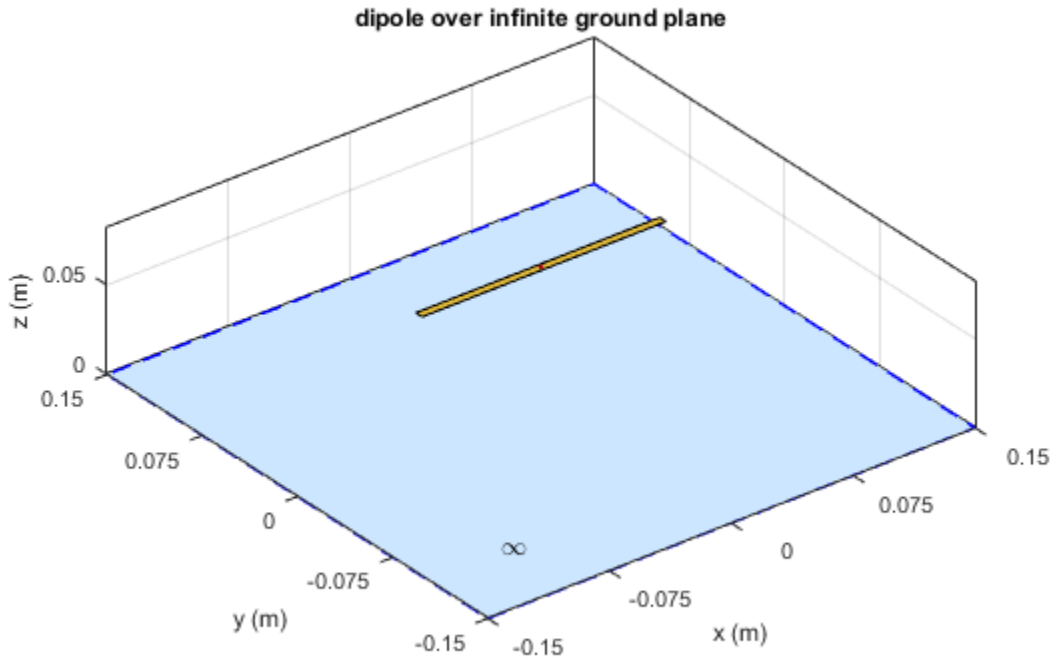
Calculate Impedance of Antenna

Calculate the impedance of reflector antenna over a freqeuncy range of 800MHz to 1GHz.

```
figure;impedance(r,800e6:5e6:1e9);
```
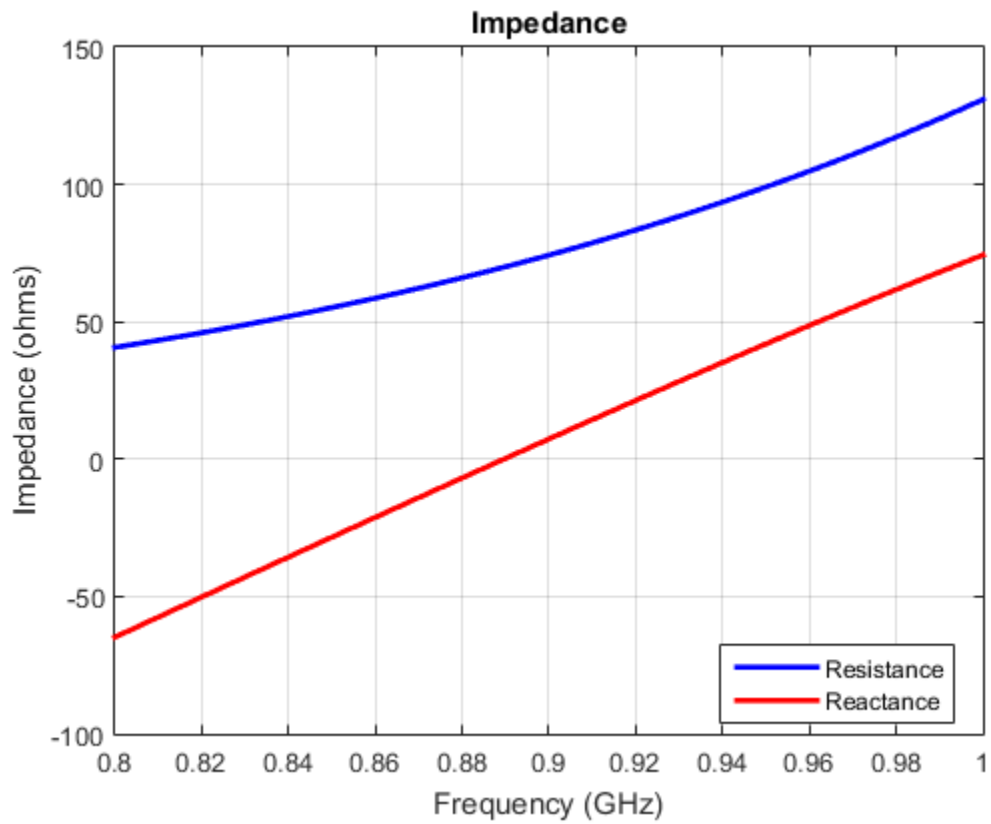
Plot Radiation Pattern of Antenna

Plot the radiation pattern of reflector antenna at a frequency of 900MHz.

```
figure;pattern(r,900e6);
```

Output : Directivity
Frequency : 900 MHz
Max value : 7.77 dBi
Min value : -124 dBi
Azimuth : [-180° , 180°]
Elevation : [-90° , 90°]
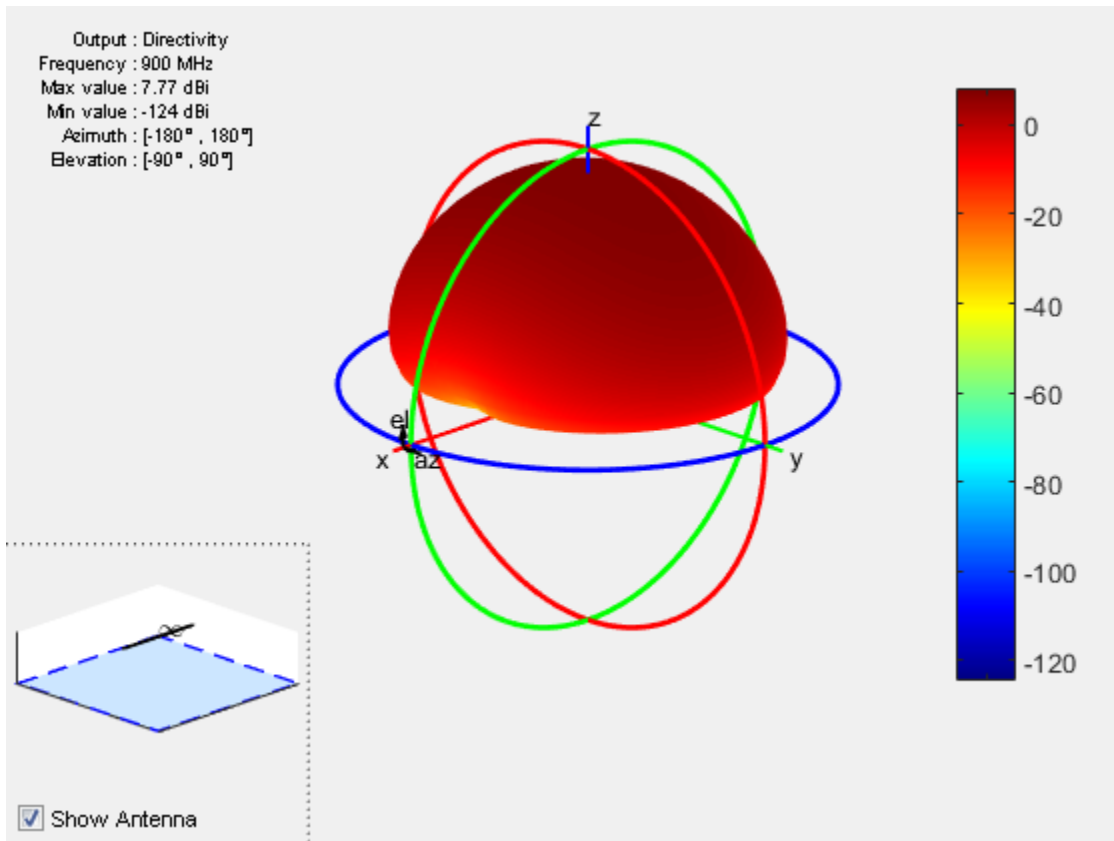
Visualize Antenna Mesh

Mesh and view the infinite ground plane reflector antenna.

```
figure; mesh(r);
```

NumTriangles : 40
MaxEdgeLength : 0.008328
MeshMode : auto

0.08

0.06

0.04

0.02

0

-0.02

-0.04

-0.06

-0.08

x (m)

z (m)

0.078
0.076
0.074

2
2

x y (m)
10³

2
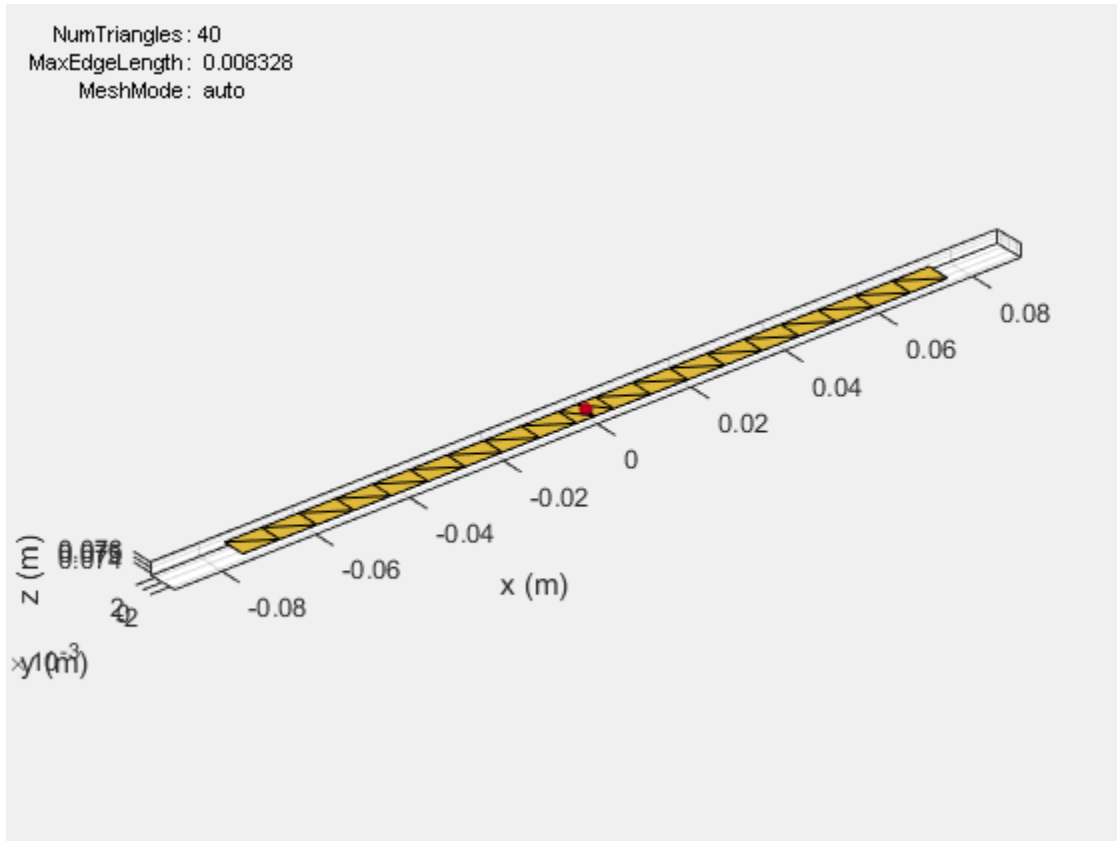
Array Concepts

Mutual Coupling

Mutual coupling is the electromagnetic interaction between the antenna elements in an array. The currents developed in each antenna element of an array depends on their own excitation and also on the contributions from adjacent antenna elements. Mutual coupling is inversely proportional to the spacing between the different antenna elements in an array. Mutual coupling in an array causes:

- Changes in the radiation pattern of the array

- Changes in the input impedance of the individual antenna elements in an array

To characterize mutual coupling, you can use mutual impedance, S-parameters, a coupling matrix, or an embedded element pattern.

Active or Scan Impedance

Active impedance, or *scan impedance*, is the input impedance of each antenna element in an array, when all elements are excited.
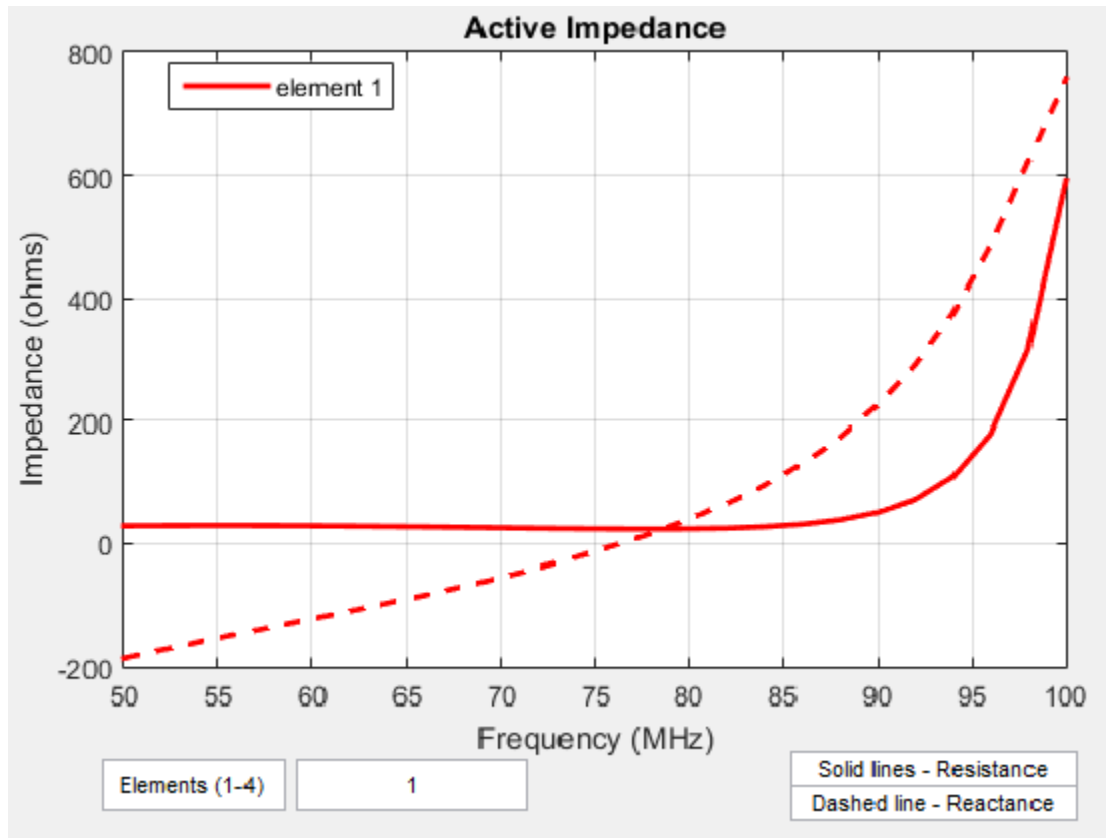
The active impedance of an array depends on:

- Array configuration
- Spacing between elements
- Phase shift applied at each element

Mutual Impedance

The effect of mutual coupling is observed or modeled by varying the space between the antenna elements in the array. Any changes in the inter-element spacings changes the *mutual impedance* between the antenna elements. For example, the plot shows the mutual impedance of a two-element dipole array as a function of inter-element spacing.
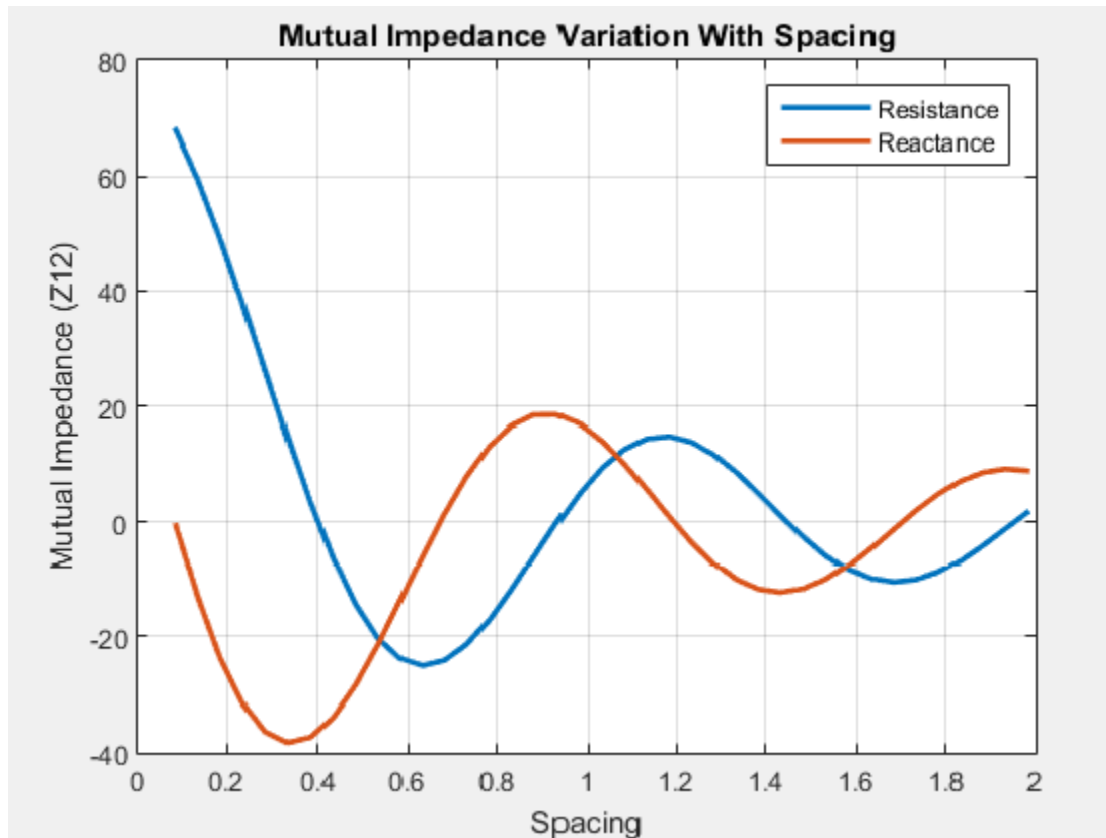
The plot shows that as the spacing between elements increases, the mutual impedance, and hence mutual coupling decreases.

Coupling Matrix

A *coupling matrix* is used to characterize the mutual coupling between the antenna elements at the port level. This matrix is calculated using S-parameters or Z-parameters and is used to decouple the array.

S-Parameter Matrix

To calculate the coupling matrix, you can use the S-parameter matrix. You calculate each column of the S-parameter matrix by feeding the antenna in that column by 1V. Consider

a element array arranged in a 2x2 grid. Visualize the grid and the element numbers using the `layout`.
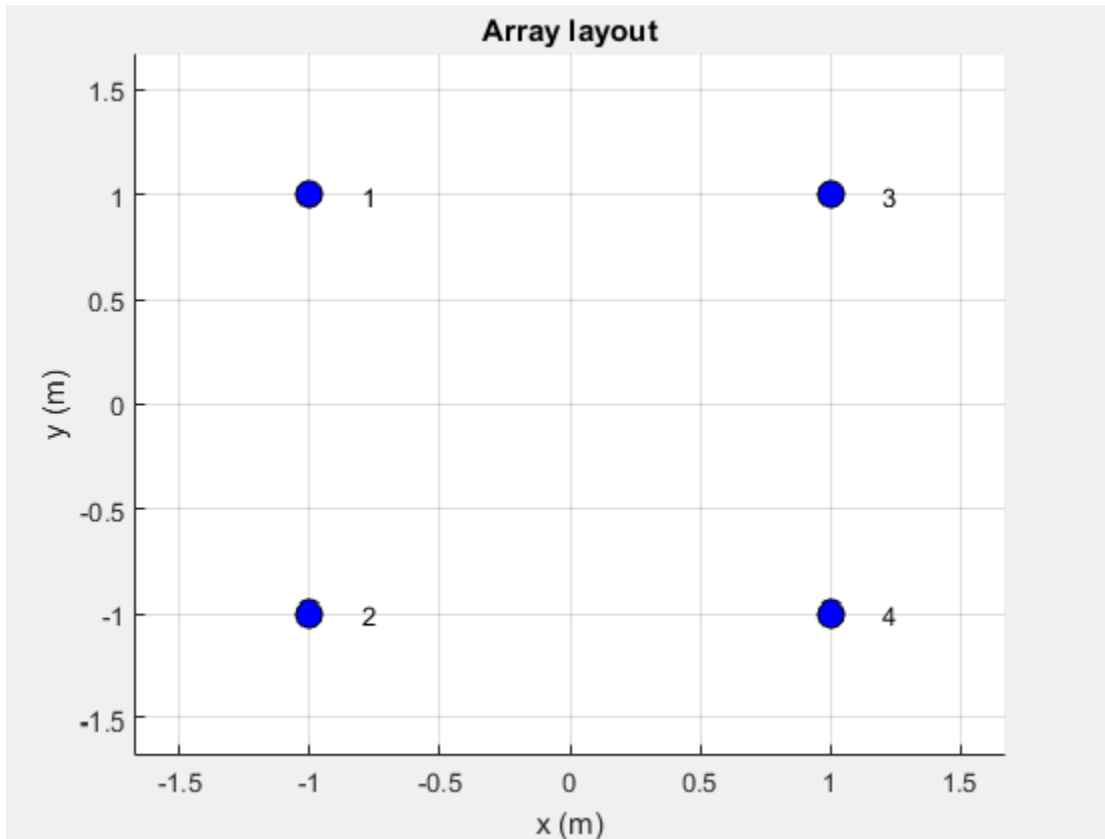
There are 4 ports in this array. The corresponding S-parameter matrix would be of size 4 x 4:

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix}$$

During this measurement, terminate rest of the antenna elements using reference resistance of 50 ohms. The termination resistance is internally set to a default of 50 ohms and can be omitted during the calculation. If the termination is different, specify the resistance value when using this function. The diagonal terms in the matrix shown above, represent the self-interaction which is also commonly referred to as the reflection coefficient. The off-diagonal terms capture the mutual coupling between the ports of the antenna.

Use `sparameters` to calculate the S-parameter coupling matrix of an array in Antenna Toolbox.

Pattern Multiplication

Pattern multiplication calculates the complete array pattern as a product of a single element pattern and array factor. Pattern multiplication assumes that all elements in an array have the same radiation pattern. The *array factor* is a function of the number of elements, the geometrical arrangement, the relative magnitude, the relative phases, and the spacings between elements in the array. This calculation is used on very large arrays of identical elements.

Isolated Element Pattern

Patterns of individual elements in small arrays vary significantly. Therefore you cannot use pattern multiplication for complete array pattern because isolated element pattern assumes that all elements have the same pattern.

You can calculate the complete array pattern of small arrays by plotting the pattern all the elements separately in a small array. To obtain this pattern, each element is individually excited and the rest of the array elements terminated using reference impedance. The plot shows the radiation pattern of individual elements of 4-element array.
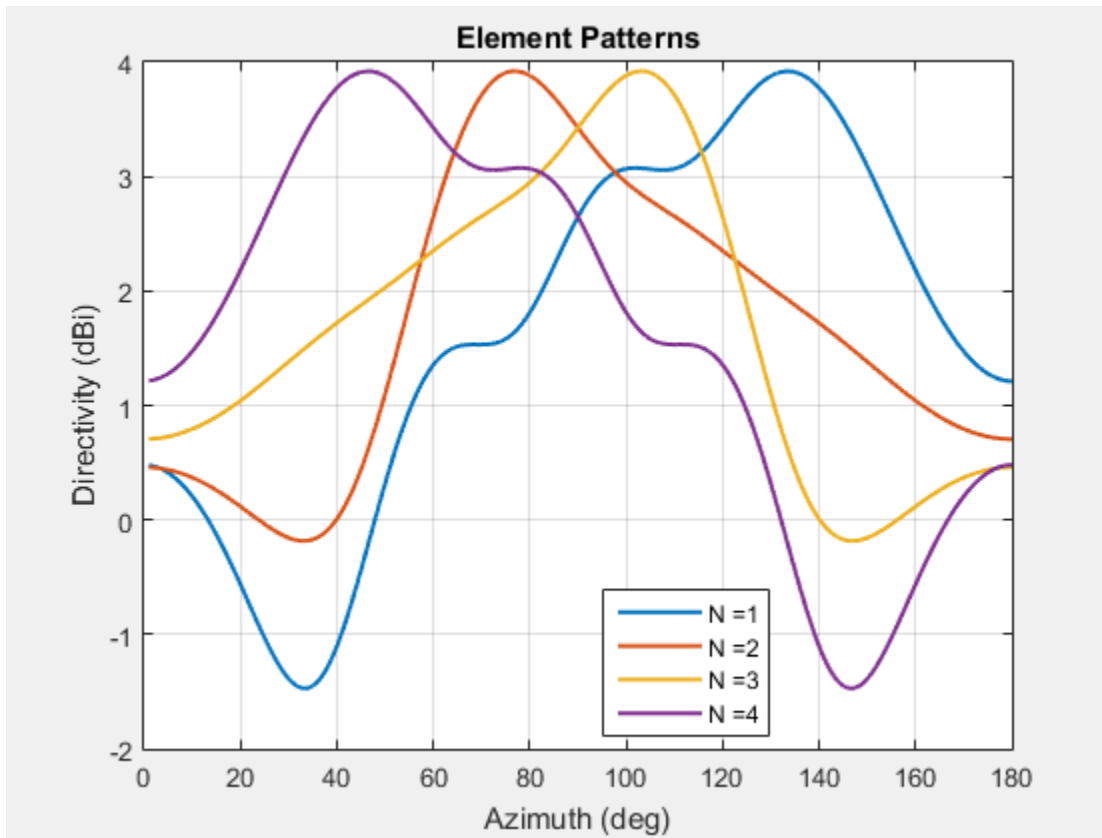
Embedded Element Pattern

The *embedded element pattern* is the pattern of a single element embedded in a finite array, calculated by driving a specific (typically the central) element in the array. The rest of the array elements are terminated using reference impedance. This method is useful for large array because the effect of mutual coupling on the individual element is captured. It is important to note that the edge effects can be ignored since the size of the array is assumed to be very large. It is common to use the center antenna element for this calculation. . Due to the size of the array the radiation patterns of the elements in the array can be approximated with the embedded element pattern instead of the isolated element pattern. Finally, pattern multiplication is used to calculate the complete array pattern.
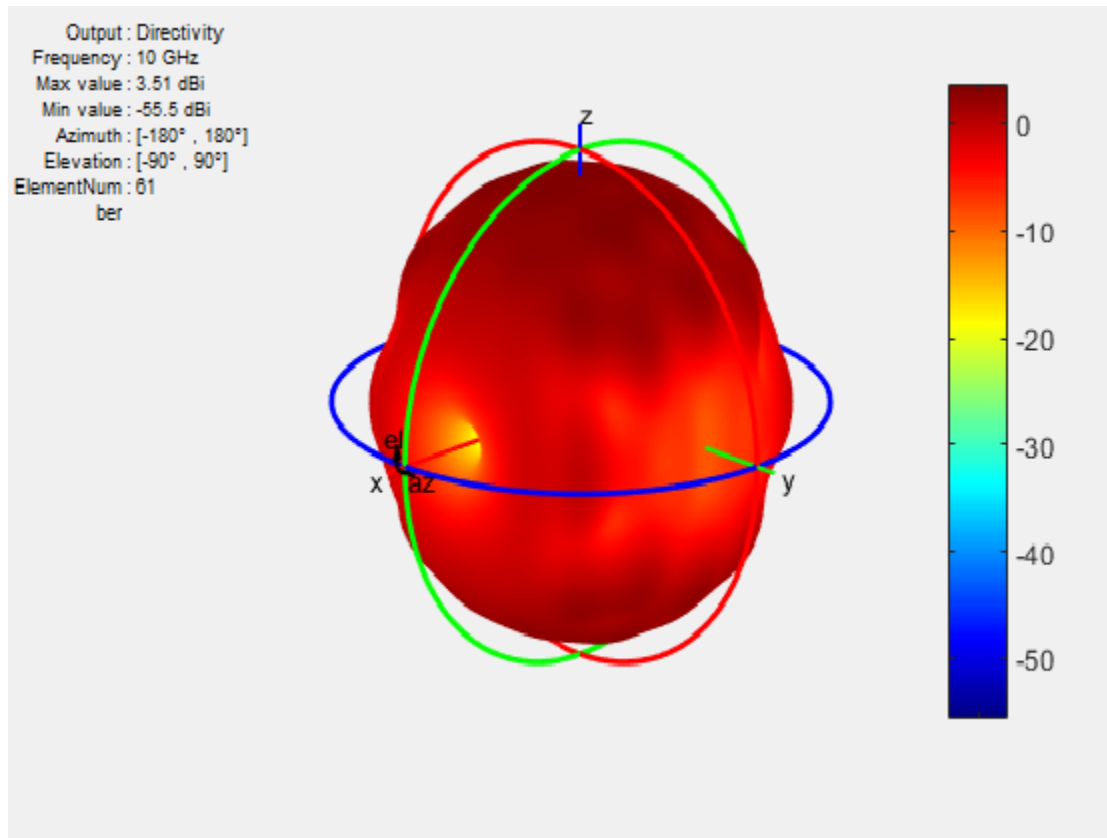
The figure above shows the embedded element pattern of a center element in a 11x11 array. You can also calculate the embedded element pattern as a magnitude of electric field.

Isolated element pattern is not recommended for large arrays as this method does not take into account the coupling effects of elements around it.

Scan Blindness

In large arrays, it is possible that the array directivity reduces drastically at certain scan angles. At these scan angles, referred to as the blind angles, the array does not radiate the power supplied at its input terminals [3]. The scan blindness can occur while using these common mechanism:

- Surface wave excitation
- Grating lobe excitation

To detect scan blindness in large finite arrays, study the embedded element pattern. In infinite array analysis, this pattern is known as the array element pattern.

References

[1] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.

Beamforming

Beamforming is the process of generating a directional beam from an antenna array. Beamforming is achieved by weighting individual elements by using sidelobe control or beam scanning.

Side Lobe Control

Side lobes are undesired and lead to reception or transmission of energy in unwanted directions. *Side lobe control* in an array is achieved using amplitude taper or amplitude weighting. Amplitude tapering changes the excitation amplitude of each element in the array. Minor lobe levels are controlled using amplitude taper that runs from the center of the array to the end of the array. Smoother amplitude tapering gives larger small side lobe levels but half-power beamwidth. These arrays are non-uniformly excited arrays.

Consider a linear array of 21 elements. Without amplitude tapering, the array contains unwanted side lobes in the desired direction:

Output : Directivity
Frequency : 70 MHz
Max value : 16.9 dBi
Min value : -19.8 dBi
Azimuth : [0° , 180°]
Elevation : 0°

By using amplitude tapering on the linear array, you control the side lobes and achieve a better main beam in the desired direction:
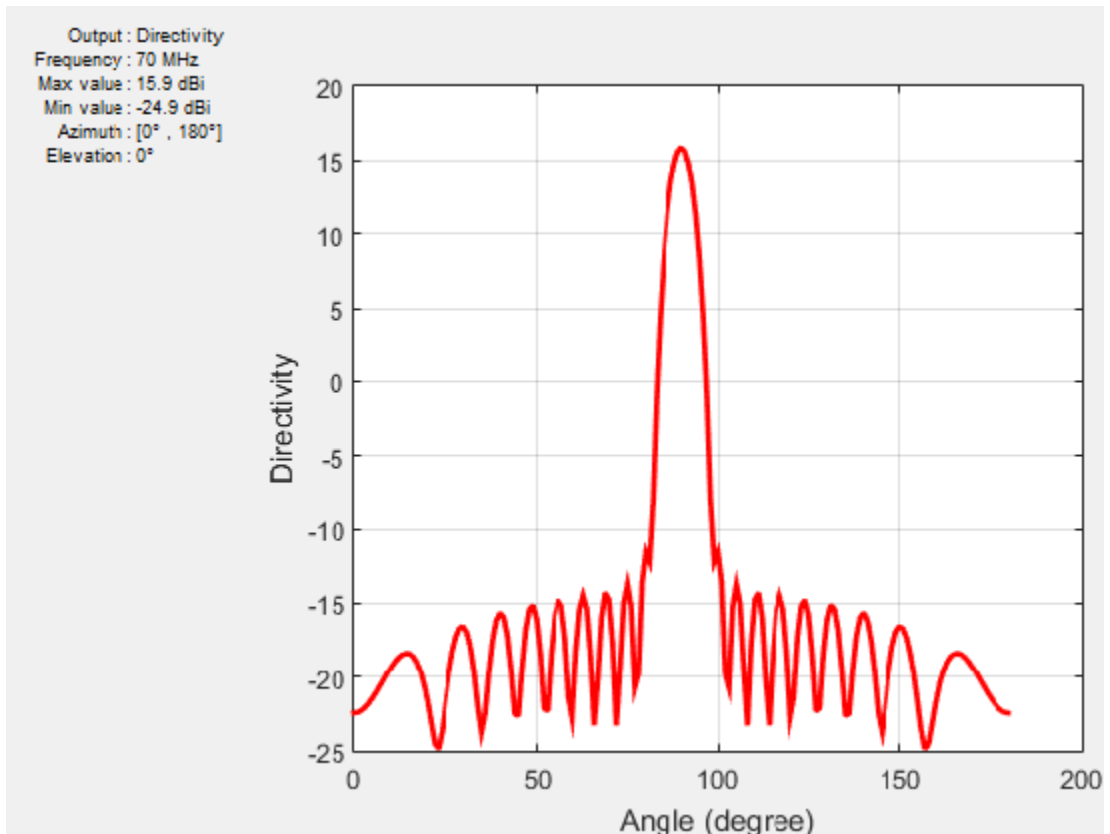
Beam Scanning

Beam scanning is the movement of a radiation pattern in space. You can achieve beam scanning by controlling the progressive phase difference between the elements to direct the beam in any desired direction. The phase shift changes the phase of the excitation currents of each element in an array. You can achieve this phase shift using phase sifter devices, a time delay, frequency scanning, beam switching, or digital beamforming.

Consider a linear array of 21 elements where peak directivity is shown without beam scanning:

Output : Directivity
Frequency : 70 MHz
Max value : 15.9 dBi
Min value : -24.9 dBi
Azimuth : [0° , 180°]
Elevation : 0°
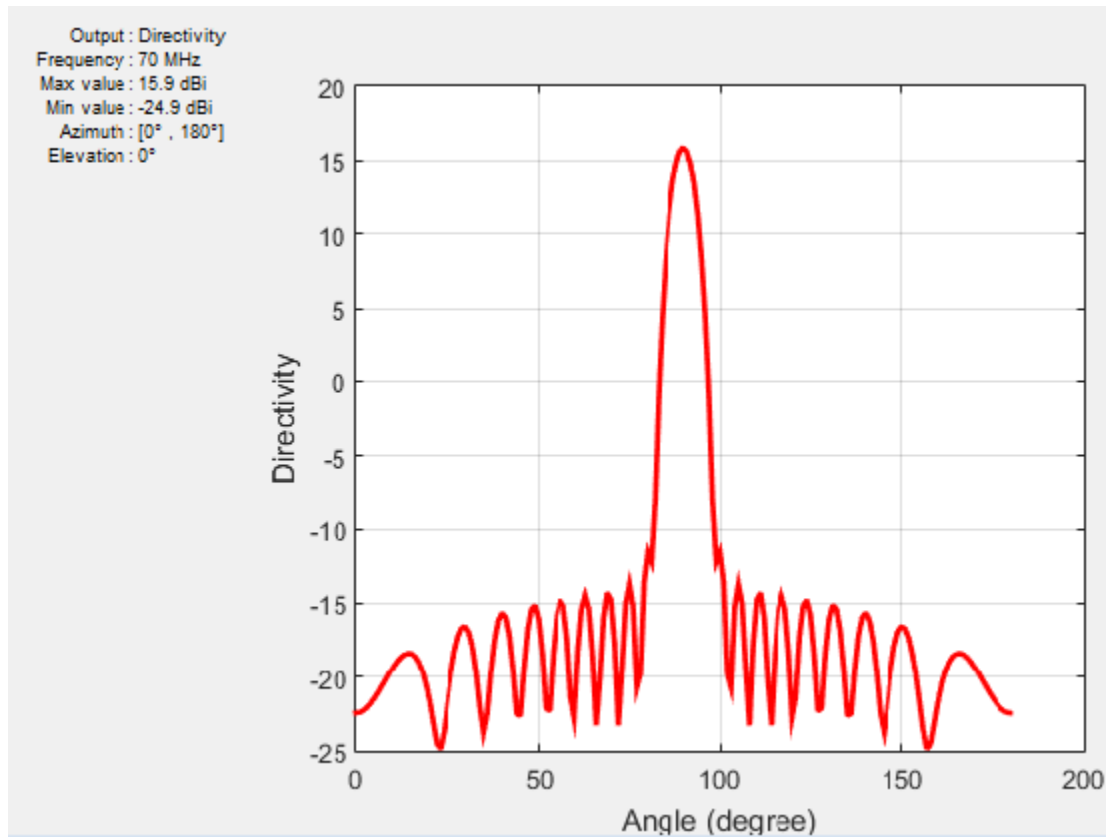
To scan the beam at a specific angle, use phase shift on the linear array:
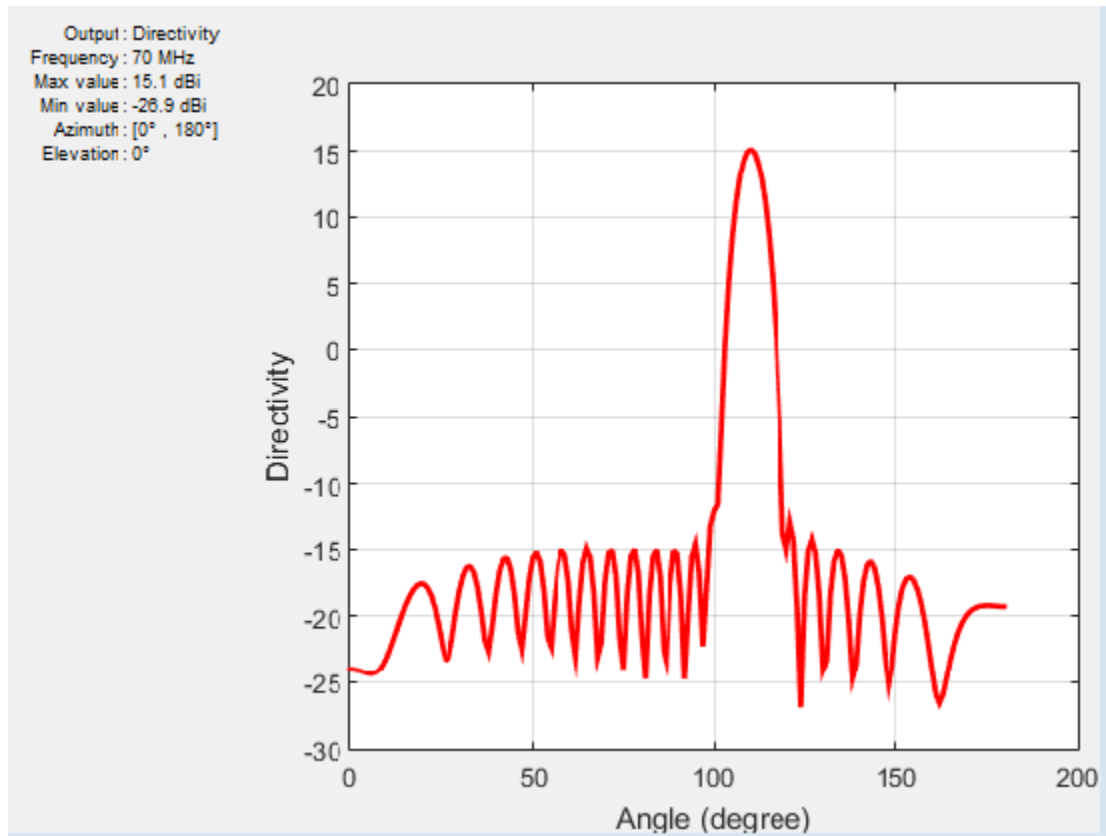
Output : Directivity
Frequency : 70 MHz
Max value : 15.1 dBi
Min value : -26.9 dBi
Azimuth : [0° , 180°]
Elevation : 0°

References

[1] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.
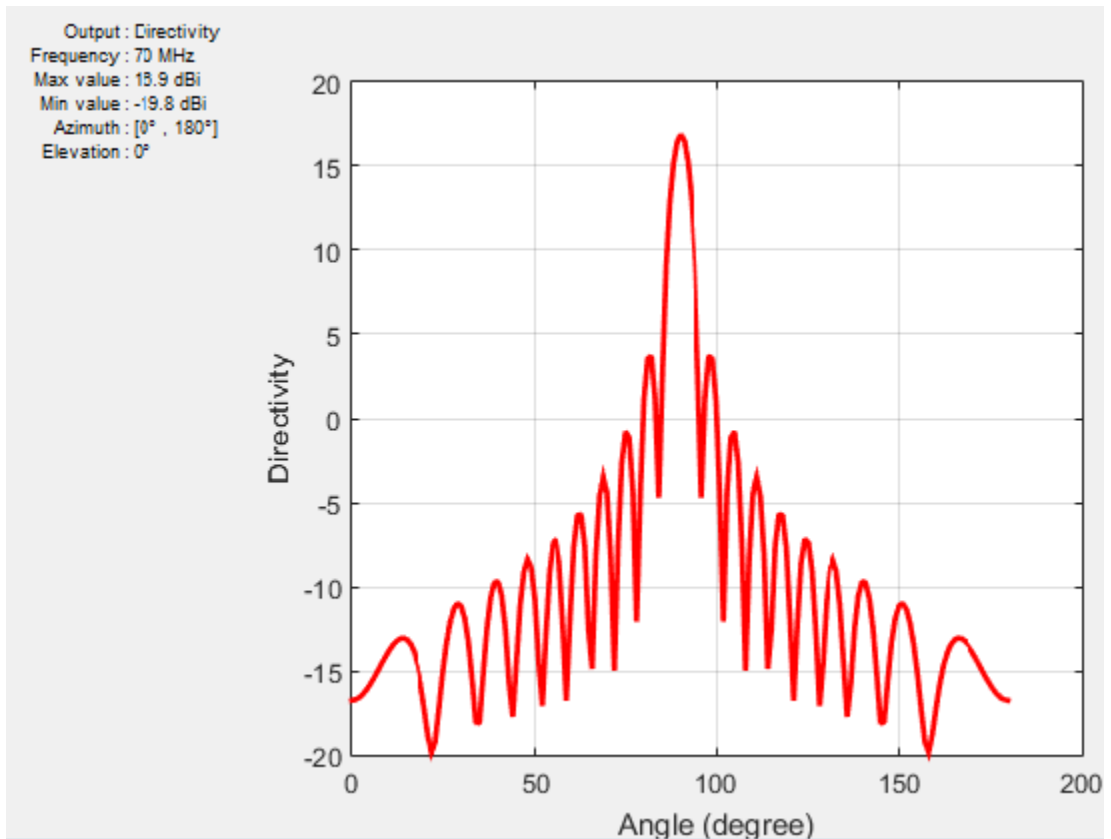
Grating Lobes

Grating lobes are the maxima of the main beam, as predicted by the pattern multiplication theorem. When the array spacing is less than or equal to $\lambda / 2$, only the main lobe exists in the visible space, with no other grating lobes. Grating lobes appear when the array spacing is greater than $\lambda / 2$. For large spacing, grating lobes can appear in the visible space even at a zero scan angle.

Consider a linear array of seven elements spaced one-half wavelength apart. This array contains no grating lobes.

Output: Directivity
Frequency: 70 MHz
Max value: 11.3 dBi
Min value: -15.2 dBi
Azimuth: [0° , 180°]
Elevation: 0°

Spacing is half-wavelength

When you increase the element spacing to one wavelength, the number of side lobes increases.

Output: Directivity
Frequency: 70 MHz
Max value: 12.7 dBi
Min value: -24.2 dBi
Azimuth: [0°, 180°]
Elevation: 0°

Spacing is one-wavelength

When you increase the element spacing to 1.5 wavelengths apart, the grating lobes appear in the visible region.

Output: Directivity
Frequency: 70 MHz
Max value: 10.5 dBi
Min value: -17.6 dBi
Azimuth: [0° , 180°]
Elevation: 0°

Spacing is one and half-wavelength
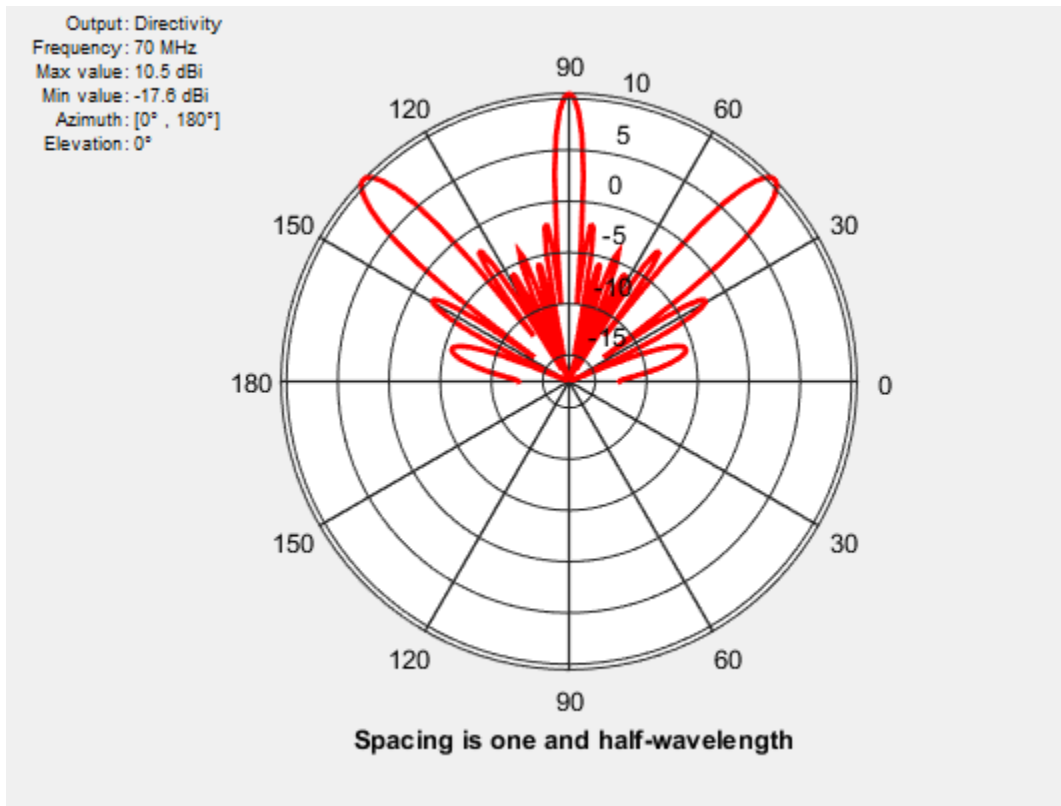
The grating lobes move in or out of the visible region depending on the spacing between the elements and the relative phase between them.

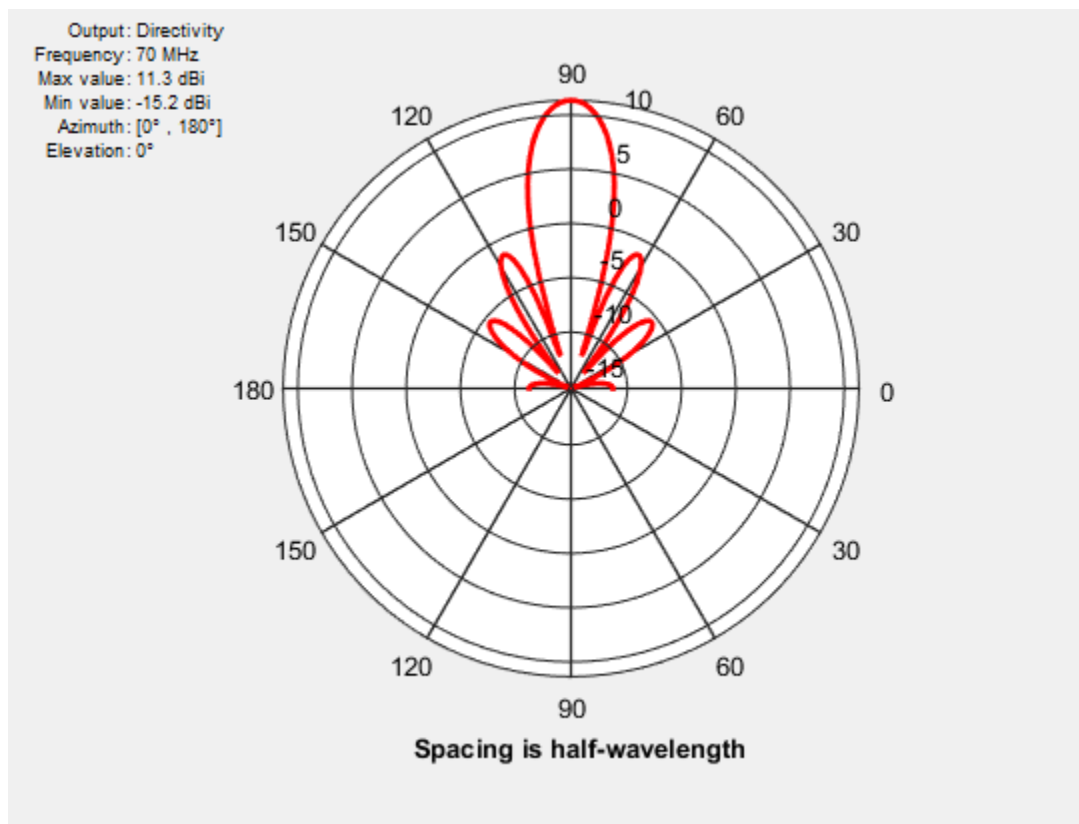## References

[1] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.

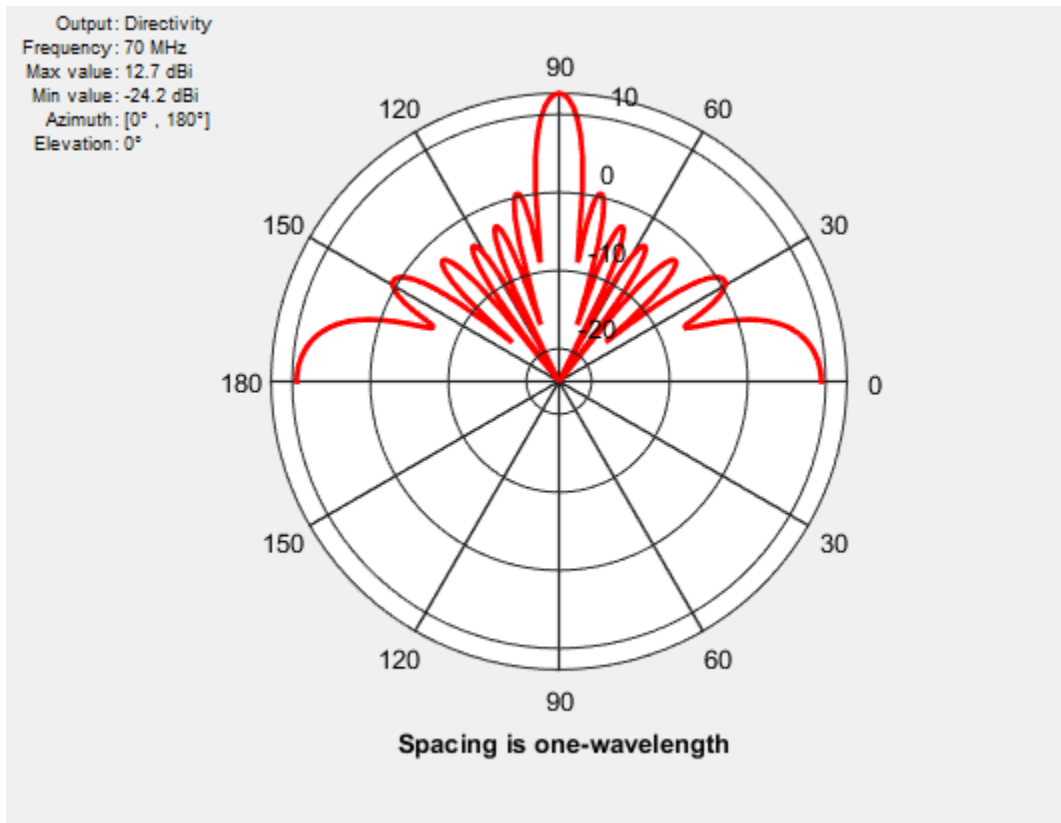# Correlation Coefficient

In this section...
"Far-Field Radiation Pattern" on page 2-18
"S-Parameter Characterization" on page 2-18

The *correlation coefficient* is the relationship between the incoming signals at the antenna ports in an array. Mutual coupling in array systems degrades the performance of the array. The correlation coefficient between antennas is used as a performance metric in multiple-input multiple-output (MIMO) systems to quantify the system performance and efficiency of the antennas. . By using the correlation coefficient, a MIMO system designer is able to understand the level of coupling that exists between the antenna ports in the system. To minimize the mutual coupling would imply to reduce the correlation coefficient between the pairs of ports. Antenna designers use two approaches to the calculate correlation coefficient: the far-field Radiation pattern and S-parameters.

Far-Field Radiation Pattern

The correlation coefficient of a two antenna array system is given by:

$$\rho_e = \frac{\left| \int\limits_{4\pi} \int [\overrightarrow{F_1}(\theta,\phi) \bullet \overrightarrow{F_2}(\theta,\phi)] d\Omega \right|^2}{\int\limits_{4\pi} \int \left|\overrightarrow{F_1}(\theta,\phi)\right|^2 d\Omega \int\limits_{4\pi} \int \left|\overrightarrow{F_2}(\theta,\phi)\right|^2 d\Omega}$$

where $\overrightarrow{F_i}(\theta,\phi)$ is the radiation pattern of the antenna system when port i is excited. To compute the correlation coefficient using this formula, you need to know the radiation pattern of the antenna and then integrate it. This approach is hard and time consuming.

S-Parameter Characterization

Antenna Toolbox™ uses the S-parameter characterization to calculate correlation between antenna elements in an array. This approach is simpler than the far-field approach because the S-parameter calculation does not use the radiation patterns of the antennas. Correlation coefficient is calculated using S-parameter by using:

$$\rho_e = \frac{\left|S_{11}^* S_{12} + S_{21}^* S_{22}\right|^2}{(1-(\left|S_{11}\right|^2 + \left|S_{21}\right|^2))(1-(\left|S_{22}\right|^2 + \left|S_{12}\right|^2))}$$

The advantages of this method are quick analysis and broadband correlation results. However, this approach assumes that the antennas are lossless and that incoming waves are uniformly distributed. To calculate and plot the correlation between antennas in an array, use the `correlation` function in Antenna Toolbox™.

References

[1] Blanch, S. Romeu, J. and Corbella, I. *Exact Representation of antenna system diversity performance from input parameter description*

[2] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.

Computational Techniques

Method of Moments

Method of Moments

The method of moments solution procedure was first applied to electromagnetic problems by Harrington. Consider a linear operator equation given by

$$\hat{A}X = Y \qquad (1)$$

where

- $\hat{A}$ is the integral operator
- Y is the known excitation function
- X is the unknown response function

In the present solver, Y is a voltage vector with a value of 1V at the feeding point and zero everywhere else. The unknown response functions to be determined are the currents on the surface of the metal. The integral operator is the impedance matrix, which calculates the interaction of the various current elements on the antenna surface.

Let X be represented by a set of known functions, termed as basis functions, or expansion functions, $(p_1, p_2, p_3, ...)$ in the domain of $\hat{A}$ as a linear combination:

$$X = \sum_{i=1}^{N} \alpha_i p_i \qquad (2)$$

where α_i are scalar constants to be determined.

In the present solver, the metal surface is divided into triangles, and the Rao-Wilton-Glisson (RWG) edge basis functions model the unknown currents on the metal surface. Substituting equation (2) into equation (1) and using the linearity of A:

$$\sum_{i=1}^{N} \alpha_i \hat{A} p_i = Y \qquad (3)$$

where equality is usually approximate. Let $(q_1, q_2, q_3, ...)$ define a set of testing functions in the range of $\hat{A}$. For the current solver, the RWG basis functions are used for testing as well. This procedure called the Galerkin procedure results in a symmetric impedance matrix.

By multiplying equation (3) with each q_i and using the linearity property of the inner product, you obtain

$$\sum_{i=1}^{N} \alpha_i \left\langle q_i, A p_i \right\rangle = \left\langle q_j, Y \right\rangle \qquad (4)$$

for $j = 1, 2, ... N$. You can solve the set of linear equations represented by equation (4) and obtain the unknown coefficients α_i's.

The simplicity of the method of moments lies in choosing the proper set of expansion and testing functions to solve your problem. This method provides a more accurate result. When applying the method of moments to complex practical problems, the solution region, is usually divided into triangular or rectangular subdomains. By using the subdomains , you can define suitable basis and testing functions and develop a general algorithm to solve the electromagnetic problem.

After the method of moments is formulated, the resulting full complex matrices have storage and computational requirements that become prohibitive as the size of the structure increases. You must be made often make trade-offs between accuracy, computational efforts and memory requirements. For complex geometries, such decisions become even more critical.

Solver Details

- The surface integral equation approach using electric field integral equation formulation with mixed potential form.
- The metal surface is meshed into triangles.
- The RWG basis functions are defined on two triangles that share a common edge on the metal surface.

- Singularity extraction is performed to accurately calculate the integrals of Green's function [3].
- The antenna is excited using a delta-gap feed model with a voltage of 1V at the feeding edge.

References

[1] Harringhton, R.F. *Field Computation by Moment Methods*, New York: Macmillan, 1968.

[2] Rao, S.M. Wilton, D.R. and Glisson, A.W. "Electromagnetic scattering by surfaces of arbitary shape", *IEEE. Trans. Antennas and Propagation*, Vol. AP-30, No. 3, May 1982, pp.409-pp.418.

[3] Wilton, D.R. Rao, S.M. Glisson, A.W. Schaubert, D.H Al-Bundak, O.M. and Butler, C.M. "Potential Integrals for uniform and linear source distribution on polygonal and polyhedral domains", *IEEE. Trans. Antennas and Propagation*, Vol. AP-30, No. 3, May 1984, pp.276-pp.281.

[4] Balanis, C.A. *Antenna Theory. Analysis and Design*, 3rd Ed. New York: Wiley, 2005.